MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXIX. JANUARY 8, 1909. No. 3

H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Samuel Butler, The Cedars, Compton, Wolverhampton;
Charles Everitt, 67 Boston Avenue, Southend-on-Sea;
J. William Gifford, Oaklands, Chard, Somerset;
James Thomas Jackson, M.A., Assoc.M.Inst.C.E., Engineering School. Trinity College, Dublin;
Dr. Arthur Scott King, Superintendent of the Physical Laboratory, Mount Wilson Observatory, California, U.S.A.;
Robert Owen, Merchant, 25 Mostyn Street, Llandudno, North Wales; and
Dr. Charles Edward St. John, Mount Wilson Observatory, California, U.S.A.,

were balloted for and duly elected Fellows of the Society.

---

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Edmund Sewall Manson, Jun., Assistant Professor of Astronomy, Ohio State University, Columbus, Ohio, U.S.A. (proposed by H. C. Lord); and
Robert Charles Slater, M.A., Assistant Master, Charterhouse, Godalming (proposed by J. Franklin-Adams).

---

Sixty presents were announced as having been received since the last meeting, including, amongst others:—

Astrographic Chart; 18 charts, presented by the Royal Observatory, Greenwich.

## *Fluctuations in the Moon's Mean Motion.*

By Simon Newcomb. (Plate 11.)

With the aid of my assistant Dr. Frank E. Ross, I have brought to a completion a study of the Moon's mean motion based on observations having an extreme range in time of about 2600 years. The data of observation are as follows:—

1. The eclipses of the Moon found in Ptolemy's Almagest, observed between B.C. 720 and A.D. 134.

2. Observations of eclipses by the Arabian astronomers, extending from 829 to 1004.

3. Observations of eclipses of the Sun and of occultations of stars by the Moon made by Gassendi, Hevelius, and others, from 1620 to 1680.

4. Observations of occultations of stars from 1670 until the present time.

The observations previous to 1750 were all worked up in my *Researches on the Motion of the Moon*, published in 1878. I have, however, subjected the results to a careful revision, and grouped them in a slightly different way from the former one. From and after 1680 the observations are of a fair degree of precision, but there are frequent gaps during the last half of the 18th century. The observations are fairly continuous since 1820.

Taken in connection with the recent exhaustive researches of Brown, which seem to be complete in determining with precision the action of every known mass of matter upon the Moon, the present study seems to prove beyond serious doubt the actuality of the large unexplained fluctuations in the Moon's mean motion to which I have called attention at various times during the past forty years. In the *Monthly Notices* for March 1903 is found a comprehensive review of the whole problem so far as it had then been developed, so that I need not enter into details at present. Indeed, the general conclusions reached in the work of 1878 have only been slightly modified in the present one.

The feature of most interest is the great fluctuation with a period of between 250 and 300 years. I call this a fluctuation rather than an inequality because, in the absence of any physical cause for its continuance, there is no reason to suppose that it will continue in the future in accordance with the law followed in the past. In the former paper I found it convenient to represent it as of the same period with the great Hansenian inequality due to the action of Venus. Singularly enough, the present research shows that the period which best represents it is still the same as that of the Hansenian inequality. In the former work I showed that the observed fluctuation could be represented by a mere change of sign of the constant term in the argument of this inequality.

Putting

$$A = 18V - 16E - g$$

Hansen's value of this term is

$$15''\cdot34 \sin (A + 30^\circ\cdot2)$$

The empirical term found by the writer in 1877 to best represent the observations was

$$-15''\cdot5 \cos A$$

The sum of these gives a term having practically the same coefficient with an argument differing little from $A - 30^\circ$. A natural suspicion would have been that an error of sign had crept into the theory. But this is disproved by the fact that the constant in question is a simple function of the longitude of the node of Venus, the relation of which to the inequality, in theory at least, admits of no doubt.

The unaccounted-for fluctuation as now found is best represented by the term

$$12''\cdot95 \sin [1^\circ\cdot31(t - 1800) + 100^\circ\cdot6]$$

The argument of the Hansenian term is

$$A = 1^\circ\cdot32\ (t - 1800) + 183^\circ\cdot9$$

Practically, the annual motion $1^\circ\cdot32$ will represent observations as well as $1^\circ\cdot31$. We may therefore write the empirical term thus—

$$\Delta\lambda = 12''\cdot95 \sin (A - 83^\circ\cdot3)$$

The former empirical term was, as above,

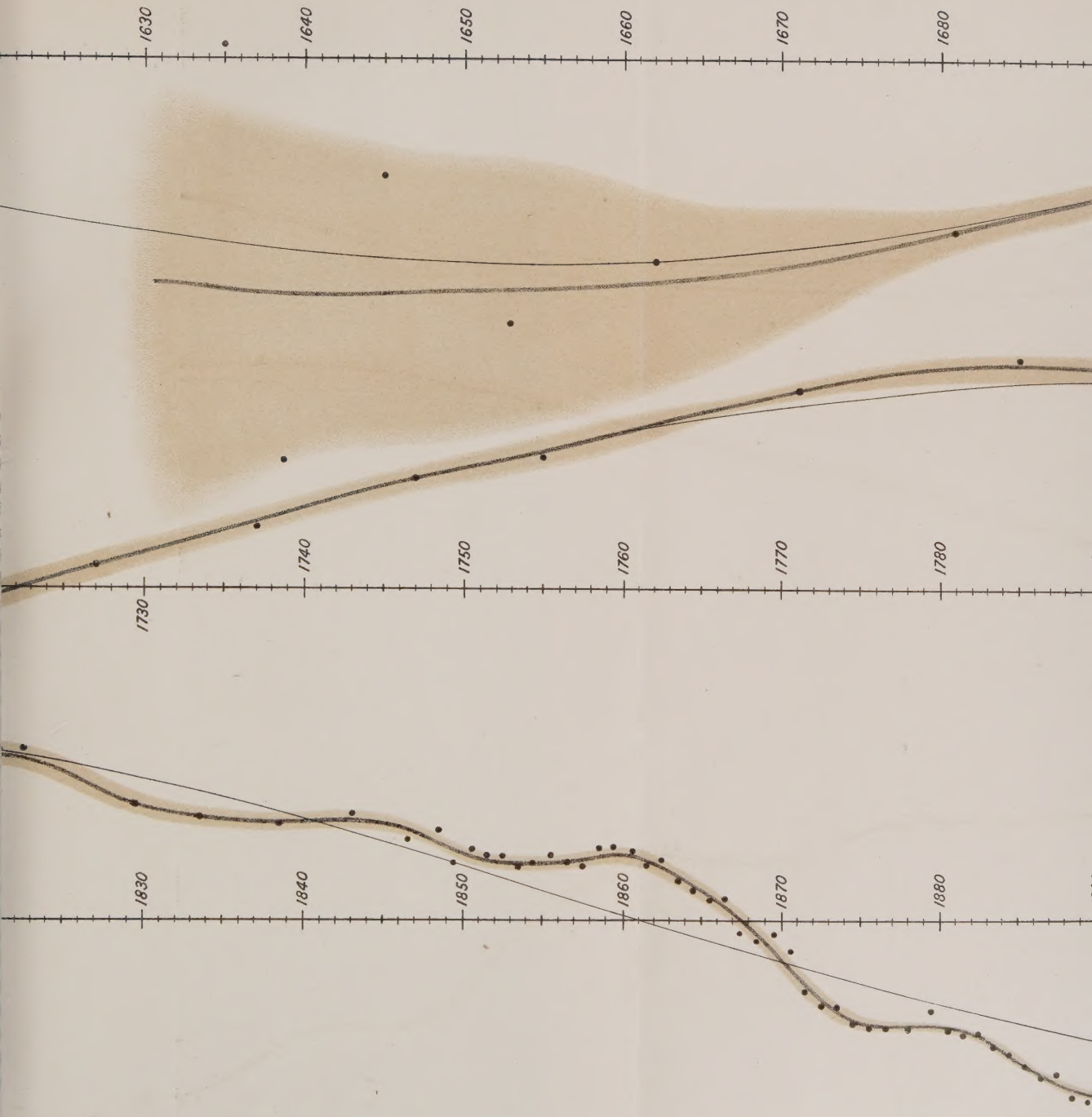
$$15''\cdot5 \sin (A - 90^\circ)$$

The following table shows the residual differences between the result of observations of the Moon since 1620 and pure gravitational theory. In deriving the elements of mean motion, it was necessary to divide the residual excess into two parts, one the great fluctuation just described, the other the smaller fluctuations which were superimposed upon it. In the table, the second column gives the minor fluctuations, which are in fact the residuals of the conditional equations. The third column shows the main fluctuation as computed from the expression given above. The sum of the two found in the fourth column, is the total excess of the Moon's observed longitude over the result of gravitational theory. It is, however, to be remarked that in this theory is included the excess of the observed over the theoretical secular acceleration.

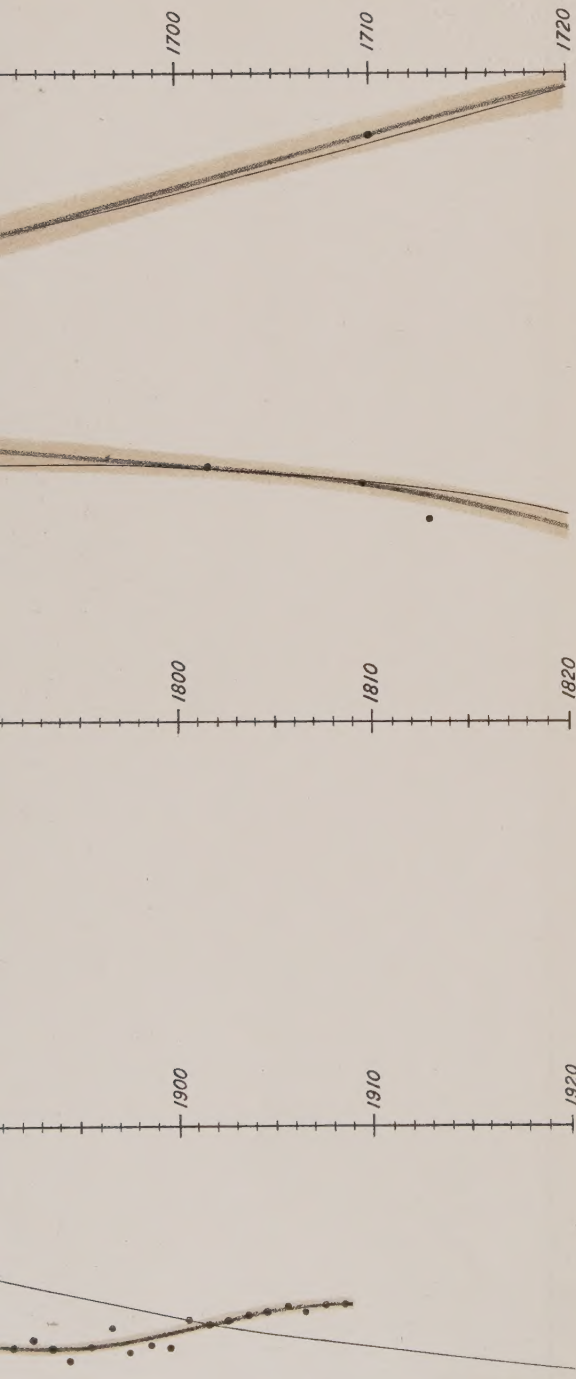
The unit of weight, as the latter is given in the last column, corresponds to a probable error of about $\pm 0''\cdot9$, and a mean error of about $\pm 1''\cdot3$. Being in many cases partly a matter of judgment, round numbers are preferred where it is doubtful. The limiting value assigned is 60, it being judged that the actual probable error can never be below $\pm 0''\cdot12$.

| Mean Date. | Minor Res. | Great Fluctuation. | Total Fluctuation. | Weight. |
|---|---|---|---|---|
| | ″ | ″ | ″ | |
| 1621 | +39 | − 9·6 | +29 | ·005 |
| 1635 | +13 | −11·7 | + 1 | ·02 |
| 1639 | −13 | −12·1 | −25 | ·04 |
| 1645 | + 5 | −12·6 | − 8 | ·03 |
| 1653 | − 4 | −12·9 | −17 | ·03 |
| 1662 | 0 | −12·7 | −13 | ·06 |
| 1681 | − 0·4 | −10·5 | −10·9 | 2 |
| 1710 | + 0·5 | − 3·7 | − 3·2 | 6 |
| 1727 | + 0·1 | + 1·2 | + 1·3 | 3 |
| 1737 | − 0·2 | + 4·1 | + 3·9 | 6 |
| 1747 | 0·0 | + 6·8 | + 6·8 | 5 |
| 1755 | − 0·3 | + 8·7 | + 8·4 | 2 |
| 1771 | + 1·4 | +11·2 | +12·6 | 9 |
| 1784·7 | + 1·6 | +12·7 | +14·3 | 5 |
| 1792 | + 0·3 | +12·9 | +13·2 | 10 |
| 1801·5 | − 0·6 | +12·7 | +12·1 | 12 |
| 1809·5 | − 0·1 | +11·9 | +11·8 | 16 |
| 1813 | − 1·2 | +11·5 | +10·3 | 16 |
| 1821 | + 0·1 | +10·3 | +10·4 | 14 |
| 1822·5 | + 0·6 | +10·0 | +10·6 | 10 |
| 1829·5 | − 1·6 | + 8·6 | + 7·0 | 20 |
| 1833·5 | − 1·6 | + 7·7 | + 6·1 | 10 |
| 1838·5 | − 0·6 | + 6·4 | + 5·8 | 30 |
| 1843 | + 1·1 | + 5·2 | + 6·3 | 20 |
| 1846·5 | + 1·1 | + 4·2 | + 5·3 | 10 |
| 1848·5 | + 1·9 | + 3·7 | + 5·6 | 8 |
| 1849·5 | + 0·1 | + 3·4 | + 3·5 | 15 |
| 1850·5 | + 1·0 | + 3·2 | + 4·2 | 18 |
| 1851·5 | + 0·8 | + 2·9 | + 3·7 | 12 |
| 1852·5 | + 0·9 | + 2·6 | + 3·5 | 8 |
| 1853·5 | + 0·7 | + 2·3 | + 3·0 | 7 |
| 1854·5 | + 1·4 | + 2·0 | + 3·4 | 14 |
| 1855·5 | + 2·1 | + 1·7 | + 3·8 | 6 |
| 1856·5 | + 1·9 | + 1·4 | + 3·3 | 6 |
| 1857·5 | + 2·1 | + 1·1 | + 3·2 | 9 |
| 1858·5 | + 3·5 | + 0·8 | + 4·3 | 8 |
| 1859·5 | + 3·9 | + 0·5 | + 4·4 | 6 |
| 1860·5 | + 3·9 | + 0·2 | + 4·1 | 12 |
| 1861·5 | + 3·3 | − 0·1 | + 3·2 | 5 |
| 1862·5 | + 3·9 | − 0·4 | + 3·5 | 7 |
| 1863·5 | + 3·0 | − 0·7 | + 2·3 | 6 |
| 1864·5 | + 2·9 | − 1·0 | + 1·9 | 10 |
| 1865·5 | + 2·6 | − 1·3 | + 1·3 | 5 |

| Mean Date. | Minor Res. | Great Fluctuation. | Total Fluctuation. |
|---|---|---|---|
| | ″ | ″ | ″ |
| 1866·5 | +2·8 | − 1·6 | + 1·2 |
| 1867·5 | +1·1 | − 1·9 | − 0·8 |
| 1868·5 | +1·0 | − 2·2 | − 1·2 |
| 1869·5 | +1·6 | − 2·5 | − 0·9 |
| 1870·5 | +0·7 | − 2·8 | − 2·1 |
| 1871·5 | −1·3 | − 3·1 | − 4·4 |
| 1872·5 | −1·8 | − 3·3 | − 5·1 |
| 1873·5 | −1·8 | − 3·6 | − 5·4 |
| 1874·5 | −2·2 | − 3·9 | − 6·1 |
| 1875·5 | −2·3 | − 4·2 | − 6·5 |
| 1876·5 | −2·1 | − 4·4 | − 6·5 |
| 1878·0 | −1·8 | − 4·8 | − 6·6 |
| 1879·5 | −0·5 | − 5·2 | − 5·7 |
| 1880·5 | −1·4 | − 5·5 | − 6·9 |
| 1881·5 | −1·6 | − 5·7 | − 7·3 |
| 1882·5 | −1·4 | − 6·0 | − 7·4 |
| 1883·5 | −2·2 | − 6·2 | − 8·4 |
| 1884·5 | −2·1 | − 6·5 | − 8·6 |
| 1885·5 | −2·5 | − 6·7 | − 9·2 |
| 1886·5 | −2·8 | − 7·0 | − 9·8 |
| 1887·5 | −2·6 | − 7·2 | − 9·8 |
| 1888·5 | −3·5 | − 7·5 | −11·0 |
| 1889·5 | −3·5 | − 7·7 | −11·2 |
| 1890·5 | −3·4 | − 8·0 | −11·4 |
| 1891·5 | −3·1 | − 8·2 | −11·3 |
| 1892·5 | −2·6 | − 8·4 | −11·0 |
| 1893·5 | −2·7 | − 8·6 | −11·3 |
| 1894·5 | −3·0 | − 8·8 | −11·8 |
| 1895·5 | −2·2 | − 9·0 | −11·2 |
| 1896·5 | −1·2 | − 9·2 | −10·4 |
| 1897·5 | −2·0 | − 9·5 | −11·5 |
| 1898·5 | −1·5 | − 9·7 | −11·2 |
| 1899·5 | −1·4 | − 9·9 | −11·3 |
| 1900·5 | 0·0 | −10·1 | −10·1 |
| 1901·5 | −0·1 | −10·3 | −10·4 |
| 1902·5 | +0·3 | −10·5 | −10·2 |
| 1903·5 | +0·6 | −10·6 | −10·0 |
| 1904·5 | +1·1 | −10·8 | − 9·7 |
| 1905·5 | +1·5 | −11·0 | − 9·5 |
| 1906·6 | +1·3 | −11·1 | − 9·8 |
| 1907·5 | +2·0 | −11·3 | − 9·3 |
| 1908·2 | +2·1 | −11·4 | − 9·3 |




UNEXPLAINED FLUCTUATIONS IN THE MOON'S MEAN LONGITUDE




West, Newman lith.

1630–1908.

The observed secular acceleration is now found to be less by 0″·37 than that which I derived in 1876. As for the theoretical value, I have added 0″·27 to the value found by Brown and myself, on account of the effect due to the combination of the Earth's oblateness with the secular diminution of the obliquity of the ecliptic. This carries the theoretical acceleration up to 6″·08. The value now found from all observations is

| | |
|---|---|
| Sec. acc. from mean equinox | 9″·07 |
| Sidereal value | 7″·96 |
| Tidal excess | 1″·88 |

The accompanying plate (Plate 11) gives a graphical representation in three sections of the residual deviations from pure theory, the motion derived from gravitational theory being represented by the straight medial lines. In order to show clearly the two parts into which the total fluctuation is divided, the term of great fluctuation is represented by a fine, sharp curve. The curve of actual longtitude is bounded on each side by a shaded area showing the mean error at each point, which is nearly $\frac{3}{2}$ of the probable error. In this way not only the fluctuations as shown by observation are exhibited, but also the error to which the curve may be subject, the probability being $\frac{2}{3}$ that at any point the true curve lies inside the shaded area, and $\frac{1}{3}$ that it lies without it.

We see by the curve as well as by the numbers, that before 1750 the observations are not sufficiently continuous, numerous, and accurate to show any fluctuation with certainty. The first minor fluctuation fairly well shown began about 1760. During the years 1765–1784 the Moon ran ahead by about 1″. Then the excess of motion ceased, and became temporarily reversed. Since 1820 the motion has been marked by rapid fluctuations, which can be so well traced on the plate that no description is necessary.

Since what is actually observed is neither the acceleration nor the speed of motion, but changes of the longitude itself, of which these quantities are respectively the second and the first derivatives as to the time, it is not possible from the observations to make any approach to an accurate estimate of the accelerating or retarding forces. The most that we can say is that these varying forces are sufficient to bring about a change of annual motion amounting to between 0″·5 and 1″·0 by acting during a period of perhaps from 4 to 6 years.

It must be remarked that the separation of the entire deviation into two parts, one the great fluctuation of long period, the other minor fluctuations superimposed upon the great one, is made merely for convenience in representing past observations, and does not imply a corresponding duplicity in the causes at play. We know that these causes have acted in a certain way in the past 250 years, but we cannot infer with confidence that they will act in the same way in the future. In other words, we cannot confidently predict

a repetition of the great fluctuation through the next 250 or 300 years. Were there no minor fluctuations whatever, the belief that the great fluctuation was permanent might have some foundation, our conclusion then being that some natural cause was in action having the period in question. But, in the actual state of things, we have no reason to believe that the close correspondence between the observed motion and the great harmonic fluctuation is more than accidental. The fact is that the variations of accelerating force necessary to produce the minor fluctuations are much greater than the forces necessary to produce the great one, the measure of this force being, not the actual deviation, but the degree of curvature at each point of the line representing the path.

The minor deviations during the past 100 years may be empirically represented by a trigonometric series, the principal term of which would have a period of 60 years, more or less, and an amplitude of perhaps 3″. But we have no reason to believe that, how accurate soever the representation may be by such a series, it will represent the future course of the Moon.

It would be of interest to compare the present curve from occultations with the deviations derived by Mr. Cowell from the Greenwich meridian observations, which he has represented by a trigonometric series. But I deem it desirable that this interesting comparison, and the conclusions to be drawn from it, should be the work of someone else.

I regard these fluctuations as the most enigmatical phenomenon presented by the celestial motions, being so difficult to account for by the action of any known causes, that we cannot but suspect them to arise from some action in nature hitherto unknown. A brief résumé of possible causes, and the difficulties in accepting them, may be attempted.

Taking it for granted that the gravitation of all known bodies has been allowed for in the comparison, and that no unknown bodies exist, the first explanation to occur to us is that the inequalities are only apparent, being perhaps due to fluctuations in the Earth's speed of rotation, and therefore in our measure of time.

I suggested this explanation in my earlier papers on the subject. It is open to the objection that it seems scarcely possible to account for changes so large as would be required through the action of known causes. But the explanation admits of an independent test from observation. If the fault is with our measure of time, it can be detected by the transits of Mercury and by the eclipses of the first satellite of Jupiter. As to the first, the discussions of the transits which I have already published, extending up to 1894, seem to preclude the possibility of any such changes in the measure of time as would account for the phenomena. The recent transit of 1907 November 14, which I have worked up in a preliminary way, seems conclusive on this point, since it would show our measure of time to be about 7 seconds slow, whereas to account for the observations of the Moon it should be more than this

amount fast. I am now engaged in the working up of observations of the first satellite of Jupiter, which may throw additional light on the subject.

A tidal friction varying with ocean currents, ice, and meteorological conditions may suggest itself. To this there is a double objection. Accepting as complete the received theory, the only effect of tidal friction would be through a tidal couple acting between the Earth and Moon. Granting the completeness of the theory, this couple could only result in the doing of work upon the rotating Earth by the Moon, and never in the Earth doing work upon the Moon, because in this case the friction would have to be a negative quantity. But apart from this, the effect of any tidal couple would be to produce wider fluctuations in the Earth's rotation, and therefore in our measure of time, than those which would by themselves account for the fluctuations in the Moon's apparent motion. At the same time it is worthy of remark that the current theory of tidal friction, and the corresponding couple, is incomplete, in that it takes no account of the tide-producing action of the Sun, which it seems quite natural to consider as incapable of modifying the lunar couple. But this should be investigated, not assumed.

The preceding suggestions seem to me to include every known cause of action. If we pass to unknown causes and inquire what is the simplest sort of action that would explain all the phenomena, the answer would be—a fluctuation in the attraction between the Earth and Moon. Accepting the law of the conservation of energy, such a fluctuation would involve an expenditure or absorption of energy somewhere in the solar system, which it seems difficult to admit. Precisely what changes of gravitation would be required I have not yet computed; but it seems quite likely that they would be below any that could be determined by experimental methods on the Earth. It would be natural to associate them with the Sun's varying magnetic activity and the varying magnetism of the Earth; but I cannot find that we have any data on this subject which would enable us to base any law upon varying magnetic action. At present I see nothing more to do than to invite the attention of investigators to this most curious subject.

One general result of the present state of things is that we cannot draw any precise conclusions from a discussion of the Moon's motion in longitude, how refined soever we make it. For example, it is impossible to derive from observation the accurate coefficient of the 18·6-year nodal inequality in longitude, owing to the varying fluctuation.

It is also not possible to predict the future motion of the Moon with precision. If we require our ephemerides of the Moon's longitude to be as exact as possible, we must correct the tabular mean longitude from time to time by observations.

*Washington:*
1908 *December* 11.

*Development of the Disturbing Function in Planetary Theory, in terms of the mean anomalies and constant elliptic elements.* By P. H. Cowell, M.A., F.R.S.

1. The following notation will be employed :—

$a$ mean distance
$f$ true anomaly
$r$ radius vector
$g$ mean anomaly
$\epsilon = \tan \frac{1}{2} \psi$ where $\sin \psi$ is the eccentricity.

Plain letters refer to the disturbed planet, accented letters to the disturbing planet.

$\omega$, $\omega'$ the distance from the intersection of the orbits to the perihelia.

$\sigma =$ sine of half the inclination of the two orbit planes.

2. Then the disturbing function (divided by the mass of the disturbing planet) is

$$R = \{r^2 + r'^2 - 2rr' \cos S\}^{-\frac{1}{2}} - \frac{r \cos S}{r'^2}$$

where $\cos S = \cos(-f + f' - \omega + \omega') - \sigma^2\{\cos(-f + f' - \omega + \omega') - \cos(f + f' + \omega + \omega')\}$

If in this expression we replace $r$ by $a$, $r'$ by $a'$, we have

$$R = \{a^2 + a'^2 - 2aa' \cos S\}^{-\frac{1}{2}} - \frac{a \cos S}{a'^2}$$

3. Suppose we have an expression

$$\phi(a) e^{If\sqrt{-1}}$$

and we require to replace $a$ by $r$ and $f$ by its value in terms of $g$, and we wish to have the modified function expressed in a series of multiples of $e^{g\sqrt{-1}}$, we may adopt the following notation :

$$D = a\frac{d}{da},$$

$$\phi(r) e^{If\sqrt{-1}} =$$

$$\sum\sum \frac{\epsilon^n}{\left(\frac{n+j}{2}\right)! \left(\frac{n-j}{2}\right)!} \{n, j, I, D\} e^{(I+j)g\sqrt{-1}}$$

where the quantity in brackets $\{n, j, I, D\}$ is an operator operating

on $\phi(\alpha)$ and the summation extends over all values of $n$, $j$ that make the quantities under the factorial sign zero or positive integers.

Now suppose we have an expression

$$\phi(a, a')e^{(\mathrm{I}f+\mathrm{I}'f')\sqrt{-1}}$$

for the expansion of

$$\phi(r, r')e^{(\mathrm{I}f+\mathrm{I}'f')\sqrt{-1}};$$

we then obtain

$$\sum\sum\sum\sum \frac{\epsilon^n \epsilon'^{n'}}{\left(\frac{n+j}{2}\right)!\left(\frac{n-j}{2}\right)!\left(\frac{n'+j'}{2}\right)!\left(\frac{n'-j'}{2}\right)!} \{n,j,\mathrm{I},\mathrm{D}.\}\{n',j',\mathrm{I}',\mathrm{D}'.\}e^{[(\mathrm{I}+j)g+(\mathrm{I}'+j')g']\sqrt{-1}}$$

the summation again extending over all values of $n, j, n', j'$ that make the quantities under the factorial signs zero or positive integers.

The advantage of employing the purely numerical denominator shown above is that the coefficients in the expanded values of $\{n, j, \mathrm{I}, x\}$ are integers.

These operators are, moreover, rational integral algebraic functions of I, $x$ of order $n$.

4. The values of $\{n, j, \mathrm{I}, x\}$, as far as $n=6$, are given in the following table.

We have $\{n. -j. \mathrm{I}. x.\} = \{n. j. -\mathrm{I}. x\}$.

$$\{0, 0, \mathrm{I}, x\} = 1$$

$$\{1, 1, \mathrm{I}, x\} = +2\mathrm{I} \qquad -x$$

$$\{2, 2, \mathrm{I}, x\} = +5\mathrm{I}+4\mathrm{I}^2-3x-4\mathrm{I}x+x^2$$

$$\{2, 0, \mathrm{I}, x\} = -4\mathrm{I}^2+x \qquad +x^2$$

5. These formulæ may be very easily developed from special cases. For instance, put $x=0$, $\mathrm{I}=1$, then we have

$$\cos(f+\theta) = \sum\sum \frac{\epsilon^n}{\left(\frac{n+j}{2}\right)!\left(\frac{n-j}{2}\right)!}\{n, j, 1, 0\}\cos(g+jg+\theta)$$

or

$$e^{f\sqrt{-1}} = \sum\sum \frac{\epsilon^n}{\left(\frac{n+j}{2}\right)!\left(\frac{n-j}{2}\right)!}\{n, j, 1, 0\}e^{(1+j)g\sqrt{-1}}$$

*Values of* $\{n.\ j.\ \mathrm{I}.\ x\}$ *from* $n=3$ *to* $n=6$.

| Power of $x$. | Power of I. | $\{3,3,\mathrm{I},x\}$ | $\{3,1,\mathrm{I},x\}$ | $\{4,4,\mathrm{I},x\}$ | $\{4,2,\mathrm{I},x\}$ | $\{4,0,\mathrm{I},x\}$ | $\{5,5,\mathrm{I},x\}$ | $\{5,3,\mathrm{I},x\}$ | $\{5,1,\mathrm{I},x\}$ | $\{6,6,\mathrm{I},x\}$ | $\{6,4,\mathrm{I},x\}$ | $\{6,2,\mathrm{I},x\}$ | $\{6,0,\mathrm{I},x\}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | +26 | − 6 | +206 | −52 | 0 | +2194 | − 570 | + 70 | +29352 | − 7728 | +1200 | 0 |
| 0 | 2 | +30 | −10 | +283 | −88 | +23 | +3360 | −1008 | +168 | +48538 | −14248 | +2650 | −280 |
| 0 | 3 | + 8 | − 8 | +120 | −60 | 0 | +1790 | − 710 | +170 | +29835 | −10600 | +2513 | 0 |
| 0 | 4 | ... | ... | + 16 | −16 | +16 | + 400 | − 240 | + 80 | + 8660 | − 4060 | +1300 | −380 |
| 0 | 5 | ... | ... | ... | ... | ... | + 32 | − 32 | + 32 | + 1200 | − 800 | + 400 | 0 |
| 0 | 6 | ... | ... | ... | ... | ... | ... | ... | ... | + 64 | − 64 | + 64 | − 64 |
| 1 | 0 | −17 | + 5 | −142 | +40 | − 6 | −1569 | + 435 | − 63 | −21576 | + 5936 | − 992 | + 60 |
| 1 | 1 | −33 | + 5 | −330 | +71 | 0 | −4080 | + 968 | −100 | −60752 | +15054 | −2136 | 0 |
| 1 | 2 | −12 | + 4 | −192 | +48 | 0 | −2995 | + 761 | − 81 | −51615 | +13375 | −1935 | + 39 |
| 1 | 3 | ... | ... | − 32 | +16 | 0 | − 840 | + 296 | − 24 | −18860 | + 5920 | − 964 | 0 |
| 1 | 4 | ... | ... | ... | ... | ... | − 80 | + 48 | − 16 | − 3120 | + 1360 | − 304 | − 48 |
| 1 | 5 | ... | ... | ... | ... | ... | ... | ... | ... | − 192 | + 128 | − 64 | 0 |
| 2 | 0 | + 9 | + 1 | + 95 | − 7 | − 9 | +1220 | − 176 | − 26 | +18694 | − 3366 | + 46 | +118 |
| 2 | 1 | + 6 | + 2 | +102 | + 3 | 0 | +1660 | − 148 | − 48 | +29535 | − 4320 | − 211 | 0 |
| 2 | 2 | ... | ... | + 24 | 0 | − 8 | + 660 | − 60 | − 36 | +15345 | − 2245 | − 255 | +195 |
| 2 | 3 | ... | ... | ... | ... | ... | + 80 | − 16 | − 16 | + 3240 | − 640 | − 136 | 0 |
| 2 | 4 | ... | ... | ... | ... | ... | ... | ... | ... | + 240 | − 80 | − 16 | + 48 |
| 3 | 0 | − 1 | − 1 | − 18 | − 6 | − 2 | − 305 | − 29 | + 13 | − 5595 | + 105 | + 197 | + 57 |
| 3 | 1 | ... | ... | − 8 | − 4 | 0 | − 230 | − 42 | + 2 | − 5530 | − 180 | + 194 | 0 |
| 3 | 2 | ... | ... | ... | ... | ... | − 40 | − 8 | + 8 | − 1680 | − 80 | + 112 | + 48 |
| 3 | 3 | ... | ... | ... | ... | ... | ... | ... | ... | − 160 | 0 | + 32 | 0 |
| 4 | 0 | ... | ... | + 1 | + 1 | + 1 | + 30 | + 14 | + 6 | + 745 | + 165 | + 9 | − 11 |
| 4 | 1 | ... | ... | ... | ... | ... | + 10 | + 6 | + 2 | + 435 | + 130 | + 17 | 0 |
| 4 | 2 | ... | ... | ... | ... | ... | ... | ... | ... | + 60 | + 20 | − 4 | − 12 |
| 5 | 0 | ... | ... | ... | ... | ... | − 1 | − 1 | − 1 | − 45 | − 25 | − 13 | − 9 |
| 5 | 1 | ... | ... | ... | .. | ... | ... | ... | ... | − 12 | − 8 | − 4 | 0 |
| 6 | 0 | ... | ... | ... | ... | ... | ... | ... | ... | + 1 | + 1 | + 1 | + 1 |

where

$\{0\ 0\ 1\ 0\} = +1$
$\{1\ 1\ 1\ 0\} = +2$
$\{1\ -1\ 1\ 0\} = -2$
$\{2\ 2\ 1\ 0\} = +9$
$\{2\ 0\ 1\ 0\} = -4$
$\{2\ -2\ 1\ 0\} = -1$
$\{3\ 3\ 1\ 0\} = +64$
$\{3\ 1\ 1\ 0\} = -24$
$\{3\ -1\ 1\ 0\} = +4$
$\{3\ -3\ 1\ 0\} = -4$
$\{4\ 4\ 1\ 0\} = +625$
$\{4\ 2\ 1\ 0\} = -216$
$\{4\ 0\ 1\ 0\} = +39$
$\{4\ -2\ 1\ 0\} = +8$
$\{4\ -4\ 1\ 0\} = -27$
$\{5\ 5\ 1\ 0\} = +7776$
$\{5\ 3\ 1\ 0\} = -2560$
$\{5\ 1\ 1\ 0\} = +520$
$\{5\ -1\ 1\ 0\} = -24$
$\{5\ -3\ 1\ 0\} = +64$
$\{5\ -5\ 1\ 0\} = -256$
$\{6\ 6\ 1\ 0\} = +117649$
$\{6\ 4\ 1\ 0\} = -37500$
$\{6\ 2\ 1\ 0\} = +8127$
$\{6\ 0\ 1\ 0\} = -724$
$\{6\ -2\ 1\ 0\} = -99$
$\{6\ -4\ 1\ 0\} = +756$
$\{6\ -6\ 1\ 0\} = -3125$

If this be regarded as a known elliptic formula, to be used as a starting-point, by raising to the I$^{th}$ power we have

$$e^{Ifv-1} = \sum\sum \frac{\epsilon^n}{\left(\frac{n+j}{2}\right)!\left(\frac{n-j}{2}\right)!} \{n, j, I, 0\} e^{(I+j)g\sqrt{-1}}$$

thus verifying all the special cases of the table obtained by putting $x = 0$.

6. Again, put $I = 0$ and we have

$$\phi(r) = \sum\sum \frac{\epsilon^n}{\left(\frac{n+j}{2}\right)!\left(\frac{n-j}{2}\right)!} \{n, j, 0. D\} \phi(a) \cos jg$$

where

$\{0\ 0\ 0\ D\} = 1$
$\{1\ 1\ 0\ D\} = -D = \{1\ -1\ 0\ D\}$
$\{2\ 2\ 0\ D\} = -3D + D^2 = \{2\ -2\ 0\ D\}$
$\{2\ 0\ 0\ D\} = +D + D^2$
$\{3\ 3\ 0\ D\} = 17D + 9D^2 - D^3 = \{3\ -3\ 0\ D\}$
$\{3\ 1\ 0\ D\} = +5D + D^2 - D^3 = \{3\ -1\ 0\ D\}$
$\{4\ 4\ 0\ D\} = -142D + 95D^2 - 18D^3 + D^4 = \{4\ -4\ 0\ D\}$
$\{4\ 2\ 0\ D\} = +40D - 7D^2 - 6D^3 + D^4 = \{4\ -2\ 0\ D\}$
$\{4\ 0\ 0\ D\} = -6D - 9D^2 - 2D^3 + D^4$
$\{5\ 5\ 0\ D\} = -1569D + 1220D^2 - 305D^3 + 30D^4 - D^5 = \{5\ -5\ 0\ D\}$
$\{5\ 3\ 0\ D\} = +435D - 176D^2 - 29D^3 + 14D^4 - D^5 = \{5\ -3\ 0\ D\}$
$\{5\ 1\ 0\ D\} = -63D - 26D^2 + 13D^3 + 6D^4 - D^5 = \{5\ -1\ 0\ D\}$
$\{6\ 6\ 0\ D\} = -21576D + 18694D^2 - 5595D^3 + 745D^4 - 45D^5 + D^6 = \{6\ -6\ 0\ D\}$
$\{6\ 4\ 0\ D\} = +5936D - 3366D^2 + 105D^3 + 165D^4 - 25D^5 + D^6 = \{6\ -4\ 0\ D\}$
$\{6\ 2\ 0\ D\} = -992D + 46D^2 + 197D^3 + 9D^4 - 13D^5 + D^6 = \{6\ -2\ 0\ D\}$
$\{6\ 0\ 0\ D\} = +60D + 118D^2 + 57D^3 - 11D^4 - 9D^5 + D^6$

but if $\log r = \log a + \rho$,

we know, by Maclaurin's Theorem, that

$$\phi(r) = \sum \frac{\rho^m}{m!} \mathrm{D}^m \phi(a).$$

Hence, in the above set of formulæ, picking out the coefficient of the first power of D, we have, on combining $\cos jg$ with $\cos -jg$,

$$\begin{aligned}
\rho = &- \epsilon \cos g \times 2\\
&- \tfrac{1}{2}\epsilon^2 \; 3\cos 2g \times 2\\
&+ \epsilon^2\\
&- \tfrac{1}{6}\epsilon^3 \; 17\cos 3g \times 2\\
&+ \tfrac{1}{2}\epsilon^3 \; 5\cos g \times 2\\
&- \tfrac{1}{24}\epsilon^4 \; 142\cos 4g \times 2\\
&+ \tfrac{1}{6}\epsilon^4 \; 40\cos 2g \times 2\\
&- \tfrac{1}{4}\epsilon^4 \, 6\\
&- \tfrac{1}{120}\epsilon^5 \; 1569\cos 5g \times 2\\
&+ \tfrac{1}{24}\epsilon^5 \; 435\cos 3g \times 2\\
&- \tfrac{1}{12}\epsilon^5 \; 63\cos g \times 2\\
&- \tfrac{1}{720}\epsilon^6 \; 21576\cos 6g \times 2\\
&+ \tfrac{1}{120}\epsilon^6 \; 5936\cos 4g \times 2\\
&- \tfrac{1}{48}\epsilon^6 \; 992\cos 2g \times 2\\
&+ \tfrac{1}{36}\epsilon^6 \, 60
\end{aligned}$$

If this is regarded as a known elliptic formula we may calculate

$$\frac{\rho^m}{m!} \text{ for } m = 2,\ 3,\ 4, \text{ etc.}$$

and thus verify the values of

$$\{n.\ j.\ \mathrm{o.\ D}\}$$

7. Taking values of $\cos f$ and $\log \frac{r}{a}$ in terms of $\epsilon$ and $g$ as known, we have now shown how to obtain

$$\{n, j, \mathrm{o, D}\} \text{ and } \{n, j, \mathrm{I, o}\}$$

We have lastly to obtain

$$\{n, j, \mathrm{I, D}\}$$

We are supposed to start from $\phi(a)\ e^{\mathrm{I}f\sqrt{-1}}$ in which we

require to replace $a$ by $r$ and $f$ by its value in terms of $g$, and we then wish to pick out the coefficient of

$$\frac{\epsilon^n}{\left(\frac{n+j}{2}\right)!\left(\frac{n-j}{2}\right)!}e^{(\mathrm{I}+j)g\sqrt{-1}}$$

We have seen how to replace $\phi(a)$ by a series whose typical term is

$$\frac{\epsilon^\nu}{\left(\frac{\nu+\gamma}{2}\right)!\left(\frac{\nu-\gamma}{2}\right)!}\{\nu.\,\gamma.\,\mathrm{o.\,D}\}e^{\gamma g\sqrt{-1}}$$

and $e^{\mathrm{I}f\sqrt{-1}}$ by a series whose typical term is

$$\frac{\epsilon^{n-\nu}}{\left(\frac{n-\nu+j-\gamma}{2}\right)!\ \left(\frac{n-\nu-j+\gamma}{2}\right)!}\{n-\nu,\ j-\gamma,\ \mathrm{I.\ o}\}e^{(\mathrm{I}+j-\gamma)g\sqrt{-1}}$$

Hence by multiplication we see that

$$\{n,j,\mathrm{I.D}\} = \sum\sum \frac{\left(\frac{n+j}{2}\right)!\left(\frac{n-j}{2}\right)!}{\left(\frac{\nu+\gamma}{2}\right)!\left(\frac{\nu-\gamma}{2}\right)!\left(\frac{n-\nu+j-\gamma}{2}\right)!\left(\frac{n-\nu-j+\gamma}{2}\right)!}\{\nu.\gamma.\mathrm{o.D}\}\{n-\nu.j-\gamma,\mathrm{I,o}\}$$

the summation extending over all values of $\nu$, $\gamma$ which make every quantity under the factorial sign in the denominator zero or a positive integer.

8. We have now merely to throw

$$\mathrm{R} = \{a^2 + a'^2 - 2aa'\cos\mathrm{S}\}^{-\frac{1}{2}} - \frac{a\cos\mathrm{S}}{a'^2}$$

into the form

$$\sum \phi(a,\ a')\cos\{\mathrm{I}f + \mathrm{I}'f' + \theta\}$$

where $\theta$, a combination of $\omega$ and $\omega'$, is a constant.

The second term of R is already in the required form, directly the value of cos S is substituted from § 2 of this paper.

We also have

$$\{a^2 + a'^2 - 2aa'\cos\mathrm{S}\}^{-\frac{1}{2}}$$

$$= \sum_{k=0}^{k=\infty}\{a^2 + a'^2 - 2aa'\cos(-f + f' - \omega + \omega')\}^{-k-\frac{1}{2}}(aa')^k\frac{(2k)!}{2^k(k!)^2}$$

$$\times\sigma^{2k}\{\cos(f + f' + \omega + \omega') - \cos(-f + f' - \omega + \omega')\}^k$$

Now if a be the larger of the two quantities $a$, $a'$, and if $\alpha$ be the ratio of the smaller to the larger,

$$\mathrm{a}\{a^2+a'^2-2aa'\cos\theta\}^{-k-\frac{1}{2}}(aa')^k \div \alpha^k$$

$$=\{1-\alpha e^{\theta\sqrt{-1}}\}^{-k-\frac{1}{2}}\{1-\alpha e^{-\theta\sqrt{-1}}\}^{-k-\frac{1}{2}}$$

$$=\left(1+\underset{1}{\frac{k+\frac{1}{2}}{}}\alpha e^{\theta\sqrt{-1}}+\frac{(k+\frac{1}{2})\cdot(k+\frac{3}{2})}{1\cdot 2}\alpha^2 e^{2\theta\sqrt{-1}}+\cdots\right)$$

$$\times\left(1+\underset{1}{\frac{k+\frac{1}{2}}{}}\alpha e^{-\theta\sqrt{-1}}+\frac{(k+\frac{1}{2})\cdot(k+\frac{3}{2})}{1\cdot 2}\alpha^2 e^{-2\theta\sqrt{-1}}+\cdots\right)$$

$$=1+\frac{(k+\frac{1}{2})^2}{1^2}\alpha^2+\frac{(k+\frac{1}{2})^2(k+\frac{3}{2})^2}{1^2\cdot 2^2}\alpha^4+\cdots$$

$$+\underset{1}{\frac{k+\frac{1}{2}}{}}\alpha\left(1+\frac{(k+\frac{1}{2})\cdot(k+\frac{3}{2})}{1\cdot 2}\alpha^2+\frac{\{(k+\frac{1}{2})(k+\frac{3}{2})\}}{1\cdot 2}\cdot\frac{\{(k+\frac{3}{2})(k+\frac{5}{2})\}}{2\cdot 3}\alpha^4+\cdots\right)(\cos\theta+\mathrm{c}$$

$$+\text{etc.}$$

$$=\sum_{i=\infty}^{i=\infty} b_{i\cdot k}\cos i\cdot\theta \quad \text{where} \quad b_{-i\cdot k}=b_{i\cdot k}$$

We now easily get

$$\mathrm{R}=\sum_{i=-\infty}^{i=\infty}\sum_{\kappa=0}^{\kappa=\infty}\sigma^{2\kappa}\mathrm{A}_{i\cdot\kappa}\cdot\cos\{i(-f+f'-\omega+\omega')+\kappa(f+f'+\omega+\omega')\}$$

where, sufficiently for a development as far as $\sigma^6$,

$$\mathrm{a}\mathrm{A}_{i\cdot 0}=\mathrm{a}\mathrm{A}_{-i\cdot 0}=b_{i\cdot 0}-\tfrac{1}{2}\sigma^2\cdot\alpha\{b_{i-1\cdot 1}+b_{i+1\cdot 1}\}$$

$$+\tfrac{3}{8}\sigma^4\alpha^2\{b_{i-2\cdot 2}+4b_{i\cdot 2}+b_{i+2\cdot 2}\}$$

$$-\tfrac{5}{16}\sigma^6\alpha^3\{b_{i-3\cdot 3}+9b_{i-1\cdot 3}+9b_{i+1\cdot 3}+b_{i+3\cdot 3}\}$$

$$\mathrm{a}\mathrm{A}_{i\cdot 1}=\alpha\, b_{i\cdot 1}-\tfrac{3}{2}\sigma^2\alpha^2(b_{i-1\cdot 2}+b_{i+1\cdot 2})$$

$$+\tfrac{15}{8}\sigma^4\alpha^3(b_{i-2\cdot 3}+3b_{i\cdot 3}+b_{i+2\cdot 3})$$

$$\mathrm{a}\mathrm{A}_{i\cdot 2}=\tfrac{3}{4}\alpha^2 b_{i\cdot 2}-\tfrac{15}{8}\sigma^2\alpha^3(b_{i-1\cdot 3}+b_{i+1\cdot 3})$$

$$\mathrm{a}\mathrm{A}_{i\cdot 3}=\tfrac{5}{8}\alpha^3 b_{i\cdot 3}.$$

On account of the second term of R we must add

$$-\frac{1}{2}\frac{a}{a'^2}(1-\sigma^2) \quad \text{to} \quad \mathrm{A}_{1\cdot 0} \quad \text{and} \quad \mathrm{A}_{-1\cdot 0}$$

and

$$-\frac{a}{a'^2} \quad \text{to} \quad \mathrm{A}_{0\cdot 1}.$$

9. Summary :—

The value of the disturbing function can therefore be expressed in the perfectly general form

$$\sum_{i=-\infty}^{i=\infty} \sum_{k=0}^{k=\infty} \Sigma_n \Sigma_j \Sigma_{n'} \Sigma_{j'}$$

$$\frac{\epsilon^n . \epsilon'^{n'}}{\left(\frac{n+j}{2}\right)! \left(\frac{n-j}{2}\right)! \left(\frac{n'+j'}{2}\right)! \left(\frac{n'-j'}{2}\right)!} \{n, j, -i+k, \mathrm{D}\}\{n', j', +i+k, \mathrm{D}'\} \mathrm{A}_{i,k}$$

$$\times \cos\{(-i+k+j)g + (i+k+j')g' + (-i+k)\omega + (i+k)\omega'\}$$

where since R is of the order $-1$ with respect to $a$, $a'$ we may replace D′ by $-1-\mathrm{D}$ if we please, and where the value of every quantity required for an expansion as far as $\epsilon^6$, $\epsilon'^6$, $\sigma^6$ (including such combinations as $\epsilon^6\epsilon'^6\sigma^6$ of the eighteenth order) is given in this paper.

10. Numerical example :—

Required the coefficient of

$$\frac{\epsilon^3\epsilon'}{2!\,1!\,1!\,1!} \cos(-2g + 4g' - 3\omega + 5\omega')$$

Here

$$i=4 \quad k=1 \quad n=3 \quad j=1 \quad n'=1 \quad j'=-1$$

The two operators are

$$\{3 . 1 . -3 . \mathrm{D}\} = +144 + 26\mathrm{D} - 5\mathrm{D}^2 - \mathrm{D}^3$$

$$\{1, -1, +5, \mathrm{D}'\} = \{1, +1, -5, \mathrm{D}'\} = -10 - \mathrm{D}' = -9 + \mathrm{D}$$

The quantity operated upon is $\sigma^2 \mathrm{A}_{4,1}$

$$= \frac{\sigma^2}{a}\left\{ a b_{4,1} - \frac{3}{2}\sigma^2 a^2 (b_{3,2} + b_{5,2}) \right\}$$

stopping at $\sigma^4\epsilon^3\epsilon^1$ or the eighth order

$$b_{4,1} = \frac{\frac{3}{2}\cdot\frac{5}{2}\cdot\frac{7}{2}\cdot\frac{9}{2}}{1\cdot 2\cdot 3\cdot 4}\alpha^4 \left\{ 1 + \frac{\frac{11}{2}}{5}\cdot\frac{\frac{3}{2}}{1}\alpha^2 + \frac{\frac{11}{2}\cdot\frac{13}{2}}{5\cdot 6}\frac{\frac{3}{2}\cdot\frac{5}{2}}{1\cdot 2}\alpha^4 + \ldots . \right.$$

$$b_{3,2} = \frac{\frac{5}{2}\cdot\frac{7}{2}\cdot\frac{9}{2}}{1\cdot 2\cdot 3}\alpha^3 \left\{ 1 + \frac{\frac{11}{2}}{4}\cdot\frac{\frac{5}{2}}{1}\alpha^2 + \ldots . \right.$$

$$b_{5,2} = \frac{\frac{5}{2}\cdot\frac{7}{2}\cdot\frac{9}{2}\cdot\frac{11}{2}\cdot\frac{13}{2}}{1\cdot 2\cdot 3\cdot 4\cdot 5}\alpha^5 \left\{ 1 + \frac{\frac{15}{2}}{6}\cdot\frac{\frac{5}{2}}{1}\alpha^2 + \ldots . \right.$$

Further, if Venus be the disturbed planet and the Earth the disturbing planet, then

$$a = 0{\cdot}723\ 331\ 5 = a$$
$$\mathfrak{a} = 1. = a'$$
$$\log \sigma = 8{\cdot}471\ 368$$
$$\omega = 234^\circ\ 7'\ 22''{\cdot}2 \qquad \epsilon = {\cdot}003\ 421\ 70$$
$$\omega' = 205^\circ\ 1'\ 29''{\cdot}1 \qquad \epsilon' = {\cdot}008\ 386\ 14$$

(see Newcomb, *A.P.*, iii. p. 406, 459)

and

$$\mathrm{D}a^m = ma^m .$$

The principal term becomes

$$-\frac{\epsilon^3\epsilon'}{2} \cdot 96 \cdot \sigma^2 a^5 \cdot \frac{\frac{3}{2}\cdot\frac{5}{2}\cdot\frac{7}{2}\cdot\frac{9}{2}}{1\cdot 2\cdot 3\cdot 4}$$

which is less than $10^{-11}$, and has still to be multiplied by the mass of the Earth.

---

*Note on Major MacMahon's paper "On the Determination of the apparent Diameter of a fixed Star."* By A. S. Eddington, M.A., M.Sc.

In Major MacMahon's paper (p. 126 *ante*), the fading of the light of a star at the instant of disappearance behind the Moon was treated purely as a problem of geometrical optics. When account is taken of the diffraction of the star's light by the limb of the Moon, his results will be found to require some modification. The argument given in this note appears to show that, judged by the rate of diminution of its light at occultation, every star will seem to have an angular diameter of about $0''{\cdot}008$, due to this diffraction.

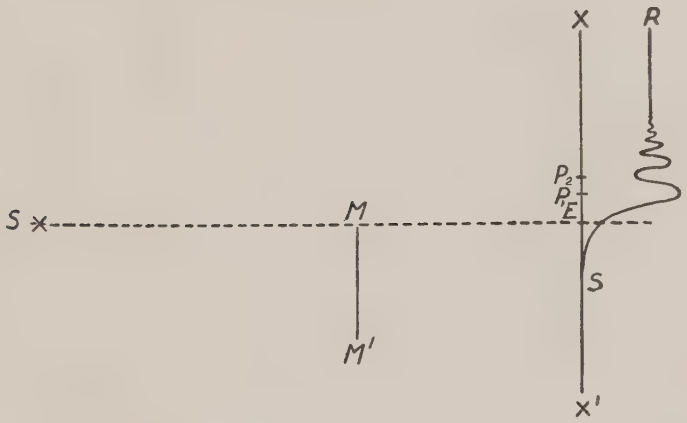
Reducing the problem to its ideally simple form, we consider a point-star S. The Moon is treated as a screen M M′ with a straight edge through M perpendicular to the plane of the paper.

Let X E X′ be a screen in the position of the observer, E being the boundary of the geometrical shadow cast by the Moon. Let J be the intensity of illumination of the screen by the star when the Moon is out of the way, the true illumination of the screen will then be as follows: * —

Inside the geometrical shadow, the illumination is zero until near the edge; then it slowly increases, reaching the value $\frac{1}{4}$ J at E; outside the geometrical shadow it still increases, until it reaches

* See, for instance, A. Schuster, *Theory of Optics*, p. 94.

a maximum of 1·37 J at $P_1$; it then decreases to a minimum of 0·78 J at $P_2$; beyond these are a series of maxima and minima, becoming rapidly less marked, and finally merging into a uniform illumination J. The curve R S shows by its varying distance from X X′ the varying intensity of the illumination along the screen. The most important part of the change is between $P_1$ and E, where the illumination changes from 1·37 J to 0·25 J. Thus if we consider the observer at the time of occultation to be carried gradually into




the shadow, the time taken to pass from the configuration S M $P_1$ to the configuration S M E will be an approximate measure of the time taken for the star to disappear, and the angle S $P_1$ M will be a measure of the spurious diameter of the star as deduced from its behaviour at occultation. Actually about $\frac{4}{3}$ S $P_1$ M should be taken as the equivalent diameter, in order to allow for the fact that the illumination at E is still rather more than a sixth of that at $P_1$.

Now $P_1$ is given by the condition

$$SM + MP_1 - SP_1 = \tfrac{3}{8}\lambda \quad \text{approximately}$$

where $\lambda$ is the wave-length of light.

Remembering that S M is almost infinitely greater than M E the condition reduces to

$$MP_1 - ME = \tfrac{3}{8}\lambda.$$

Therefore $$1 - \cos P_1ME = \tfrac{3}{8}\frac{\lambda}{MP_1}$$

$$\therefore \quad \angle P_1ME = \sqrt{\tfrac{3}{4}\frac{\lambda}{MP_1}}.$$

Filling in the values

$$\lambda \text{ (photographic)} = 4{\cdot}3 \times 10^{-5} \text{ cms.}$$

$$MP_1 = 380000 \text{ kilometres.}$$

We find

$$\angle SP_1M = \angle P_1ME = 0''{\cdot}006.$$

Further,

$$P_1E = 36 \text{ feet.}$$

The spurious occultation diameter due to the diffraction of light at the Moon's limb is thus about $0''{\cdot}008$.

In laboratory diffraction problems it is usually rather difficult to determine what is the effect of using a telescope for direct observation of the diffraction pattern instead of studying the illumination of a screen, because the aperture of the telescope occupies a space which would be covered by a great many diffraction bands. In the present case, however, the difficulty does not arise: $P_1E$ being 36 feet, the aperture of the telescope is very small compared with the width of a diffraction band; and consequently, when the telescope is pointed directly to the star, the change of phase across the aperture (the crinkling of the wave-front) is negligible. The telescope therefore receives plane waves from the star, the intensity varying according to the law we have already considered. Thus the star is seen in its normal position, its brightness varying in the same way as the illumination on a hypothetical screen, coinciding with the telescope. Of course the star will have the ordinary spurious apparent diameter, depending on the aperture of the telescope, very much greater than, and independent of, the spurious occultation diameter which we have been considering. It may seem surprising that the only effect of diffraction is to alter the apparent brightness of the star, for in the laboratory the effect is to border the obstacle with bright and dark bands. These are not seen in the astronomical problem, because, being of the order $''{\cdot}006$ in width, the resolving power of the telescope is far too small to distinguish them.

It is hardly necessary to consider in detail the other respects in which the actual problem differs from one of diffraction by a straight edge. The fact that the Moon is a globe, and not a flat disk, may affect the results, and so also may the irregularities of its contour. (The first Fresnel zone at the Moon's surface is about 42 feet in radius.) The fact that the precise effect of diffraction cannot be certainly estimated would rather add to the dificulties of applying the proposed method. But whether this possibility is serious or not, the normal diffraction diameter of $0''{\cdot}008$ is sufficiently great to mask the true diameter in all but very exceptional cases.

*Some Remarks on Lambert's Theorem.* By H. C. Plummer, M.A.

1. It is a curious fact that the proofs of a theorem so celebrated as Lambert's, as they appear in at least the majority of text-books, are very unsatisfactory in the treatment given to the ambiguities of sign which must occur in the statement of the theorem. In most of the cases which arise in practice the arc of the orbit is comparatively short, and there is really no question about the signs to be chosen. This, coupled with the fact that it was commonly supposed that the form of the theorem was in general unsuitable for use in the determination of orbits, has apparently caused a certain carelessness in the demonstration. Recent work has, however, tended to place the value of the theorem more clearly in evidence.* For the case of elliptic motion the essence of the matter in regard to the distinction in the ambiguous signs has been given by Callandreau.† For hyperbolic motion under attraction to a centre a simple proof was given by Adams ‡ in terms of hyperbolic functions, but the question of signs was ignored by him altogether. The case of motion under a repulsive force along the convex branch of an hyperbola does not seem to have been discussed at all, though the motion of matter in a comet's tail may give this case a real practical importance. It may therefore be useful to give here some examination to the methods of proof in the several cases, with such remarks as may be suggested incidentally.

2. The case of elliptic motion will be discussed first. Let $E_1$, $E_2$ be the eccentric anomalies of the starting and end points respectively, and let $E_2 > E_1$, both angles being less than 360°. In the first instance we suppose that the motion is in the same sense as the anomaly increases. It is easy to prove that, $\mu$ being the force at unit distance,

$$\mu^{\frac{1}{2}} a^{-\frac{3}{2}} t = \epsilon - \delta - (\sin\epsilon - \sin\delta)$$

where

$$\sin^2 \tfrac{1}{2}\epsilon = (r_1 + r_2 + c)/4a \equiv m_1^2, \quad \sin^2 \tfrac{1}{2}\delta = (r_1 + r_2 - c)/4a \equiv m_2^2$$

The relations of $\epsilon$ and $\delta$ to the eccentric anomalies are given by

$$\epsilon - \delta = E_2 - E_1, \quad \cos \tfrac{1}{2}(\epsilon + \delta) = e \cos \tfrac{1}{2}(E_2 + E_1)$$

The angle $\frac{1}{2}(\epsilon + \delta)$ is merely defined by the latter relation, and may be taken between the limits 0° and 180°; and $\frac{1}{2}(\epsilon - \delta)$ must lie between the same limits. Hence $\epsilon$ must lie between 0° and 360°, while $\delta$ must lie between $\pm$ 180°. It follows that $\sin \frac{1}{2}\epsilon$ and $\cos \frac{1}{2}\delta$ are both positive. If, then, $\epsilon_1$ and $\delta_1$ are the principal values resulting from

$$\sin \tfrac{1}{2}\epsilon = +m_1, \sin \tfrac{1}{2}\delta = +m_2$$

* *Cf. M.N.*, lxiii. p. 147.
† *Annales de l'Obs. de Paris, Mémoires*, vol. xxiii.
‡ *Collected Scientific Papers*, vol. i. p. 411.

we have to choose between $\epsilon_1$ and $2\pi - \epsilon_1$ and between $\pm\delta_1$. Now the intercept $x$ on the major axis between the chord joining the end points and the centre is given by

$$x = a \sec \tfrac{1}{2}(E_2 + E_1) \cos \tfrac{1}{2}(E_2 - E_1)$$

or

$$(x + ae)/(x - a) = \{\cos \tfrac{1}{2}(E_2 - E_1) + e \cos \tfrac{1}{2}(E_2 + E_1)\} \div \{\cos \tfrac{1}{2}(E_2 - E_1) - \cos \tfrac{1}{2}(E_2 + E_1)\}$$
$$= \cos \tfrac{1}{2}\epsilon \cos \tfrac{1}{2}\delta / \sin \tfrac{1}{2}E_1 \sin \tfrac{1}{2}E_2$$

Under the conditions imposed, the sign of $\cos \frac{1}{2}\epsilon$ is the same as that of the fraction on the left: this shows that $\epsilon = \epsilon_1$ or $2\pi - \epsilon_1$ according as the chord intersects the axis outside or inside the range between the second focus and "perihelion." Similarly

$$(x - ae)/(x - a) = \sin \tfrac{1}{2}\epsilon \sin \tfrac{1}{2}\delta / \sin \tfrac{1}{2}E_2 \sin \tfrac{1}{2}E_1$$

which shows that $\delta = \delta_1$ or $-\delta_1$ according as the chord does not, or does, intersect the axis between the attracting focus and "perihelion."

3. Let now

$$\epsilon_1 - \delta_1 - (\sin \epsilon_1 - \sin \delta_1) = \mu^{\frac{1}{2}} a^{-\frac{3}{2}} T_1 = nT_1$$
$$\epsilon_1 + \delta_1 - (\sin \epsilon_1 + \sin \delta_1) = \mu^{\frac{1}{2}} a^{-\frac{3}{2}} T_2 = nT_2$$

Then, under the conditions assumed, we have found that

$$t = T_1 , \ 2\pi/n - T_2 \text{ or } 2\pi/n - T_1$$

according as the segment of which the curved boundary is described in the motion contains (1) no focus, (2) the empty but not the attracting focus, (3) both foci. But the conditions have precluded motion past "perihelion," and this condition must be removed. It is merely necessary to consider the motion over the complementary parts of the ellipse corresponding to the cases already distinguished. We thus find that

$$t = 2\pi/n - T_1 , \ T_2 \text{ or } T_1$$

according as the segment contains (1) both foci, (2) the attracting but not the empty focus, (3) no focus. And now, by combining the six cases considered separately, we arrive at the final result for motion within a single circuit:

(1) $t = T_1$ when the segment of which the curved boundary is described contains neither focus.
(2) $t = 2\pi/n - T_1$ when the segment contains both foci.
(3) $t = T_2$ when the segment contains the attracting but not the other focus.
(4) $t = 2\pi/n - T_2$ when the segment contains the second but not the attracting focus.

4. If we are given merely the position of the attracting focus and of the two points 1, 2 on the orbit and the value of $a$, it is

easy to see that all the four cases enumerated are possible solutions, and that it is not possible to discriminate between them without further knowledge of the circumstances. For the second focus lies on the circle whose centre is 1 and radius $2a - r_1$; it also lies on the circle whose centre is 2 and radius $2a - r_2$. The necessary and sufficient condition that these circles shall intersect is clearly

$$2a - r_1 + 2a - r_2 > c$$

*i.e.* $$4a > r_1 + r_2 + c$$

$c$ being the length of the chord joining 1, 2.

If the condition (which, we may notice, entails the reality of $\epsilon$ and $\delta$) be fulfilled, the ellipse will be real, and there are two positions which the second focus may occupy, one on either side of the chord joining 1, 2. Hence in the one case the foci are on the same side, in the other case on opposite sides of the chord. In the former we have case (1) or (2) according to the arc chosen in passing between 1 and 2; in the latter we have case (3) or (4) according as the motion takes place *via* "perihelion," or *via* "aphelion." The four cases are thus all possible, and the choice among them must be guided by additional knowledge. The four solutions are given by Gauss,* but he did not discuss the geometrical criteria which serve to distinguish them. Curiously enough, Callandreau himself omitted case (3), while the solutions given in the text-books are (1) and (3). When $4a = r_1 + r_2 + c$ the circles referred to above touch one another at a point between 1 and 2. There is then only one position of the second focus and the chord 12 passes through it. The four solutions become equal in pairs ($\epsilon = \pi$), as they also do when the attracting focus lies on the chord ($r_1 + r_2 = c$; $\delta = 0$).

5. Some difficulty arises in the calculation of $t$ when $\epsilon$ and $\delta$ are small or nearly equal. The former case occurs when $a$ is large, *i.e.* when the ellipse approaches parabolic eccentricity; the latter occurs when the chord $c$ is small. Both these circumstances may concur. For elliptic motion I have previously† given a modification of the formula which avoids any uncertainty in the computation, and obviates resort to any special tables.

6. The formulæ for parabolic motion can be deduced from those already obtained for the ellipse. As the second focus approaches an infinite distance it cannot lie within the finite segment whose boundary is described in the motion. Hence we are limited to the cases (1) and (3) of § 3; $\epsilon$ has its principal value and $\delta$ is positive or negative according as the angular motion about the centre of attraction is less or greater than 180°. Hence in the limit

$$a\epsilon^2 = r_1 + r_2 + c, \quad a\delta^2 = r_1 + r_2 - c$$
$$6\mu^{\frac{1}{2}}t = a^{\frac{3}{2}}(\epsilon^3 - \delta^3)$$
$$= (r_1 + r_2 + c)^{\frac{3}{2}} \mp (r_1 + r_2 - c)^{\frac{3}{2}}$$

* *Theoria Motus*, § 107.

† *M.N.*, lxvi. p. 496.

the upper sign being taken when the segment does not contain the focus, the lower when it does contain the focus. This is Euler's equation, the use of which is much facilitated by Encke's transformation and auxiliary tables. This transformation is itself not free from ambiguities, and the question has been recently discussed by S. Tscherny.*

7. We consider next the case of motion along the concave branch of an hyperbola. Any point on one branch of an hyperbola referred to its centre as origin can be represented by

$$x = a\cosh u\,,\quad y = a(e^2-1)^{\frac{1}{2}}\sinh u$$

The focus for which the branch is concave is $(ae\,,\,0)$. Hence

$$r^2 = a^2\{(e-\cosh u)^2 + (e^2-1)(\cosh^2 u - 1)\}$$
$$r = a(e\cosh u - 1)$$

and

$$\begin{aligned} c^2 &= a^2(\cosh u_2 - \cosh u_1)^2 + a^2(e^2-1)(\sinh u_2 - \sinh u_1)^2 \\ &= 4a^2\sinh^2\tfrac{1}{2}(u_2-u_1)\{-1+e^2\cosh^2\tfrac{1}{2}(u_2+u_1)\} \\ &= 4a^2\sinh^2\alpha\sinh^2\beta \end{aligned}$$

where

$$\alpha = \tfrac{1}{2}(u_2-u_1)\,,\quad \cosh\beta = e\cosh\tfrac{1}{2}(u_2+u_1)$$

We may suppose $u_2 > u_1$ and therefore $\alpha$ positive; also of the two possible values of $\beta$, the positive can be chosen. Again

$$r_2 + r_1 = 2a(\cosh\alpha\cosh\beta - 1)$$

If then

$$\epsilon = \beta+\alpha\,,\quad \delta = \beta-\alpha$$

$$(r_1+r_2+c)/4a \equiv m_1{}^2 = \sinh^2\tfrac{1}{2}\epsilon\,,\quad (r_1+r_2-c)/4a \equiv m_2{}^2 = \sinh^2\tfrac{1}{2}\delta$$

Now

$$\begin{aligned} ht &= \int r^2 d\theta = \int(x\,dy - y\,dx) \\ &= a^2(e^2-1)^{\frac{1}{2}}\int\{(\cosh u - e)\cosh u - \sinh^2 u\}du \\ &= a^2(e^2-1)^{\frac{1}{2}}\{(u_1-u_2) - e(\sinh u_1 - \sinh u_2)\} \\ &= a^2(e^2-1)^{\frac{1}{2}}(-2\alpha + 2\sinh\alpha\cosh\beta) \\ &= a^2(e^2-1)^{\frac{1}{2}}\{-(\epsilon-\delta) + (\sinh\epsilon - \sinh\delta)\} \end{aligned}$$

where

$$h^2 = \mu a(e^2-1)$$

$\mu$ being the attracting force at unit distance. Hence

$$\mu^{\frac{1}{2}}a^{-\frac{3}{2}}\,t = \sinh\epsilon - \sinh\delta - (\epsilon-\delta)$$

where $\epsilon$ and $\delta$ are given by

$$\sinh\tfrac{1}{2}\epsilon = \pm m_1\,,\quad \sinh\tfrac{1}{2}\delta = \pm m_2$$

or

$$\exp.(\tfrac{1}{2}\epsilon) = \pm m_1 + (m_1{}^2+1)^{\frac{1}{2}}\,,\quad \exp.(\tfrac{1}{2}\delta) = \pm m_2 + (m_2+1)^{\frac{1}{2}}$$

* *A.N.*, No. 4280.

and it is necessary to decide between these pairs of positive, reciprocal values. Now $\epsilon$ is positive because $\alpha$ and $\beta$ are both positive. Hence exp. $(\frac{1}{2}\epsilon) > 1$ and $+m_1$ must be chosen. Again, the equation of the chord gives for the intercept on the axis beyond the focus

$$\begin{aligned} x - ae &= a \operatorname{sech} \tfrac{1}{2}(u_2+u_1)\{\cosh \tfrac{1}{2}(u_2-u_1) - e \cosh \tfrac{1}{2}(u_2+u_1)\} \\ &= -2a \operatorname{sech} \tfrac{1}{2}(u_2+u_1) \sinh \tfrac{1}{2}\epsilon \sinh \tfrac{1}{2}\delta \end{aligned}$$

Hence, according as the chord intersects the axis beyond the focus or not, *i.e.* according as the angular motion is greater or less than 180°, $\delta$ is negative or positive, that is, $-m_2$ or $+m_2$ must be chosen. Accordingly

$$\begin{aligned} \mu^{\frac{1}{2}} a^{-\frac{3}{2}} t = &+ 2m_1(m_1^2+1)^{\frac{1}{2}} \mp 2m_2(m_2^2+1)^{\frac{1}{2}} \\ &- 2 \operatorname{Log}\{(m_1^2+1)^{\frac{1}{2}} + m_1\} \pm 2 \operatorname{Log}\{(m_2^2+1) + m_2\} \end{aligned}$$

where Log denotes natural logarithm, and the upper or lower sign is to be taken according as the angular motion about the attracting focus is less or greater than 180°.

8. With any given (positive) value of $a$ it is always possible to draw two hyperbolas such that the points 1 and 2 lie on the branch which is concave to the attracting centre. For the position of the second focus is determined by the intersections of two circles whose centres are at 1 and 2, and whose radii are $2a+r_1$ and $2a+r_2$ respectively. The intersections are always real, for the necessary condition

$$2a + r_1 + 2a + r_2 > c$$

is always fulfilled. They lie on opposite sides of the chord 12, and hence one focus must be on the same side as the attracting focus, the other on the opposite side of the chord. These facts correspond with the ambiguity of sign in the solution, which can only be removed, therefore, by independent knowledge of the geometrical circumstances of the motion.

9. To obtain a form more suitable for calculation, we put

$$m_1^2 = \sinh^2 \tfrac{1}{2}\epsilon = \tan^2 \mu_1 , \quad m_2^2 = \sinh^2 \tfrac{1}{2}\delta = \tan^2 \mu_2$$

where $\mu_1$ and $\mu_2$ are to be taken in the first quadrant. Then we have

$$\begin{aligned} \tfrac{1}{2}\mu^{\frac{1}{2}} a^{-\frac{3}{2}} t = &\tan \mu_1 \sec \mu_1 \mp \tan \mu_2 \sec \mu_2 \\ &- \operatorname{Log} \tan (45^\circ + \tfrac{1}{2}\ 1) \pm \operatorname{Log} \tan (45^\circ + \tfrac{1}{2}\mu_2) \end{aligned}$$

In what follows only the upper signs will be retained. We can write

$$\begin{aligned} \mu^{\frac{1}{2}} a^{-\frac{3}{2}} t &= \sinh \epsilon - \sinh \delta - 2 \sinh \tfrac{1}{2}(\epsilon - \delta) \\ &\quad + 2\{\sinh \tfrac{1}{2}(\epsilon-\delta) - \tfrac{1}{2}(\epsilon-\delta)\} \\ &= 4 \sinh \tfrac{1}{2}(\epsilon-\delta) \sinh^2 \tfrac{1}{4}(\epsilon+\delta) + \tfrac{1}{3} \sinh^3 \tfrac{1}{2}(\epsilon-\delta). \text{A} \end{aligned}$$

where

$$A = 6\{\sinh \tfrac{1}{2}(\epsilon-\delta) - \tfrac{1}{2}(\epsilon-\delta)\}/\sinh^3 \tfrac{1}{2}(\epsilon-\delta)$$

and evidently tends to unity as $(\epsilon-\delta)$ becomes smaller. Now

$$c/2a = m_1^2 - m_2^2 = \sinh \tfrac{1}{2}(\epsilon-\delta)\sinh \tfrac{1}{2}(\epsilon+\delta)$$

and it can be found that, as a sufficient approximation when $(\epsilon-\delta)$ is small,

$$\log A = 3{\cdot}6 \log \operatorname{sech} \tfrac{1}{4}(\epsilon-\delta)$$

Hence

$$\mu^{\frac{1}{2}}t = a^{\frac{1}{2}}c \tanh \tfrac{1}{4}(\epsilon+\delta) + \tfrac{1}{2^{\frac{1}{4}}}a^{-\frac{3}{2}}c^3 \sinh^{-3} \tfrac{1}{2}(\epsilon+\delta)\{\operatorname{sech} \tfrac{1}{4}(\epsilon-\delta)\}^{3{\cdot}6}$$

and we easily find that

$$\begin{aligned}
&\tanh \tfrac{1}{4}(\epsilon+\delta) = \sin \tfrac{1}{2}(\mu_1+\mu_2)/\cos \tfrac{1}{2}(\mu_1-\mu_2)\\
&\sinh \tfrac{1}{2}(\epsilon+\delta) = 2 \sin \tfrac{1}{2}(\mu_1+\mu_2) \cos \tfrac{1}{2}(\mu_1-\mu_2) \sec \mu_1 \sec \mu_2\\
&\operatorname{sech} \tfrac{1}{4}(\epsilon-\delta) = (\cos \mu_1 \cos \mu_2)^{\frac{1}{2}} \sec \tfrac{1}{2}(\mu_1+\mu_2)
\end{aligned}$$

The equation has thus been reduced to a form which can present no difficulty to the computer when the angles $\mu_1$ and $\mu_2$ become small or nearly equal, as they do when the hyperbola is nearly parabolic or the chord is small.

10. It has been found useful to employ the approximation

$$\begin{aligned}
A &\equiv 6\{\sinh \tfrac{1}{2}(\epsilon-\delta) - \tfrac{1}{2}(\epsilon-\delta)\}/ \sinh^3 \tfrac{1}{2}(\epsilon-\delta)\\
&= \{\operatorname{sech} \tfrac{1}{4}(\epsilon-\delta)\}^{3{\cdot}6}
\end{aligned}$$

and it will be well to examine the nature of this approximation. We have

$$\begin{aligned}
\operatorname{Log}(\sinh x - x) &= \operatorname{Log}(-i \sin ix - x)\\
&= \operatorname{Log}\left(\frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\right)\\
&= 3 \operatorname{Log} x - \operatorname{Log} 6 + S_1
\end{aligned}$$

where $i^2 = -1$ and *

$$S_1 = \frac{x^2}{20} - \frac{x^4}{16800} - \frac{x^6}{756000} + \ldots$$

But

$$\operatorname{Log}\sinh x = \operatorname{Log} x + \frac{x^2}{6} - \frac{x^4}{180} + \frac{x^6}{2835} - \ldots$$

Hence writing $x$ for $\frac{1}{2}(\epsilon-\delta)$ we find

$$\operatorname{Log} A = -\frac{9}{20}x^2 + \frac{93}{5600}x^4 - \frac{89}{84000}x^6 \ldots$$

* Cf. *A.N.*, No. 3883, p. 300

Now

$$\text{Log cosh } x = \frac{x^2}{2} - \frac{x^4}{12} + \frac{x^6}{45} - \ \ldots$$

so that

$$3{\cdot}6 \text{ Log cosh } \tfrac{1}{2}x = \frac{9}{20}x^2 - \frac{3}{160}x^4 + \frac{1}{800}x^6 \ \ldots$$

which shows that

$$\text{Log A} - 3{\cdot}6 \text{ Log sech } \tfrac{1}{2}x = -\frac{3}{1400}x^4 + \frac{1}{5250}x^6 \ \ldots$$

The residual terms only begin to be significant even in 7-figure logarithms when $x$ is about 0·1, *i.e.* when $\epsilon - \delta$ is 0·2; and when the difference between $\epsilon$ and $\delta$ is so great there is no difficulty in the direct calculation of $t$. And it must be noted that the term of which A is a factor is itself small in comparison with the other term, and therefore A is not required with very great accuracy.

11. The method of § 7 can now be applied to the case of motion on the convex branch. The focus for which the same branch of the hyperbola is convex is at $(-ae, 0)$. In this case

$$r = a(e \cosh u + 1)$$

and, as before,

$$c = 2a \sinh \alpha \sinh \beta$$

where

$$\alpha = \tfrac{1}{2}(u_2 - u_1), \ \cosh \beta = e \cosh \tfrac{1}{2}(u_2 + u_1)$$

and $\alpha$ and $\beta$ are both positive. If then

$$\epsilon = \beta + \alpha, \ \delta = \beta - \alpha$$

since

$$r_2 + r_1 = 2a(\cosh \alpha \cosh \beta + 1)$$

it follows that

$$m_1^2 \equiv (r_1 + r_2 + c)/4a = \cosh^2 \tfrac{1}{2}\epsilon, \ m_2^2 \equiv (r_1 + r_2 - c)/4a = \cosh^2 \tfrac{1}{2}\delta$$

Now

$$\begin{aligned} ht &= a^2(e^2 - 1)^{\frac{1}{2}} \int \{(\cosh u + e) \cosh u - \sinh^2 u\} du \\ &= a^2(e^2 - 1)^{\frac{1}{2}} \{(u_2 - u_1) + e(\sinh u_2 - \sinh u_1)\} \\ &= a^2(e^2 - 1)^{\frac{1}{2}} \{\epsilon - \delta + \sinh \epsilon - \sinh \delta\} \end{aligned}$$

where

$$h^2 = \mu a(e^2 - 1)$$

$\mu$ being the repulsive force at unit distance. Hence

$$\mu^{\frac{1}{2}} a^{-\frac{3}{2}} t = \epsilon - \delta + \sinh \epsilon - \sinh \delta$$

where $\epsilon$ and $\delta$ are given by

$$\cosh \tfrac{1}{2}\epsilon = +m_1, \ \cosh \tfrac{1}{2}\delta = +m_2$$

These are necessarily positive quantities, but two solutions are again possible in each case for

$$\exp.(\tfrac{1}{2}\epsilon) = m_1 \pm (m_1^2 - 1)^{\frac{1}{2}},\ \exp.(\tfrac{1}{2}\delta) = m_2 \pm (m_2^2 - 1)^{\frac{1}{2}}$$

these being pairs of positive, reciprocal values. As before, $\epsilon$ is positive, and therefore exp. $(\frac{1}{2}\epsilon) > 1$, and the upper sign must be taken in the first ambiguity. Also the intercept on the axis beyond the focus for which the branch is concave, in this case the empty focus, is as before

$$x - ae = -2a \operatorname{sech} \tfrac{1}{2}(u_2 + u_1) \sinh \tfrac{1}{2}\epsilon \sinh \tfrac{1}{2}\delta$$

which shows that $\delta$ is positive, *i.e.* exp. $(\frac{1}{2}\delta) > 1$, if $x - ae$ is negative, that is, if the angular motion about the empty focus is less than 180°. Hence

$$\mu^{\frac{1}{2}}a^{-\frac{3}{2}}t = 2 \operatorname{Log}\{m_1^2 - 1)^{\frac{1}{2}} + m_1\} \mp 2 \operatorname{Log}\{m_2^2 - 1)^{\frac{1}{2}} + m_2\} + 2m_1(m_1^2 - 1)^{\frac{1}{2}} \mp 2m_2(m_2^2 - 1)^{\frac{1}{2}}$$

where the upper or lower sign is to be taken according as the angular motion about the empty focus is less or greater than 180°.

12. The second focus of the hyperbola in this case is one of the intersections of the circles whose centres are at 1 and 2, and whose radii are $r_1 - 2a$ and $r_2 - 2a$ respectively. In order that these circles may intersect it is necessary that

$$r_1 - 2a + r_2 - 2a > c$$

*i.e.*

$$4a < r_1 + r_2 - c$$

If this condition, which entails the reality of $\epsilon$ and $\delta$, be fulfilled, two hyperbolas can be drawn so that the two points lie on the branch convex to the focus of repulsion. As in other cases, the two positions which the second focus may occupy are on opposite sides of the chord, and therefore in the one case it is on the same side as the first focus, in the other case on the opposite side. The ambiguity in the solution is inherent in the data.

13. To reduce the last equation to a form more suitable for calculation we put

$$m_1^2 - 1 = \tan^2 \mu_1,\ m_2^2 - 1 = \tan^2 \mu_2$$

where $\mu_1$ and $\mu_2$ are to be taken in the first quadrant. Then we have

$$\tfrac{1}{2}\mu^{\frac{1}{2}}a^{-\frac{3}{2}}t = \tan \mu_1 \sec \mu_1 \mp \tan \mu_2 \sec \mu_2 + \operatorname{Log} \tan(45^\circ + \tfrac{1}{2}\mu_1) \mp \operatorname{Log} \tan(45^\circ + \tfrac{1}{2}\mu_2)$$

The lower signs give a sum of four positive terms which cannot present much difficulty to calculate, and the case is not of practical importance. Retaining only the upper signs we find since

$$c/2a = m_1^2 - m_2^2 = \sin(\mu_1 - \mu_2) \sin(\mu_1 + \mu_2) \sec^2 \mu_1 \sec^2 \mu_2$$

that

$$\tan \mu_1 \sec \mu_1 - \tan \mu_2 \sec \mu_2 = c/4a.(1 + \sin \mu_1 \sin \mu_2) \div \sin \tfrac{1}{2}(\mu_1 + \mu_2) \cos \tfrac{1}{2}(\mu_1 - \mu_2)$$

and

$$\begin{aligned} &\operatorname{Log} \tan (45^\circ + \tfrac{1}{2}\mu_1) - \operatorname{Log} \tan (45^\circ + \tfrac{1}{2}\mu_2) \\ &= \operatorname{Log}\{\cos \tfrac{1}{2}(\mu_1 + \mu_2) + \sin \tfrac{1}{2}(\mu_1 - \mu_2)\} - \operatorname{Log}\{\cos \tfrac{1}{2}(\mu_1 + \mu_2) - \sin \tfrac{1}{2}(\mu_1 - \mu_2)\} \\ &= \operatorname{Log}(1 + x) - \operatorname{Log}(1 - x) \end{aligned}$$

where

$$\begin{aligned} x &= \sin (\mu_1 - \mu_2)/2 \cos \tfrac{1}{2}(\mu + \mu_2) \cos \tfrac{1}{2}(\mu_1 - \mu_2) \\ &= c/4a \,.\, \cos^2 \mu_1 \cos^2 \mu_2 / \sin (\mu_1 + \mu_2) \cos \tfrac{1}{2}(\mu_1 + \mu_2) \cos \tfrac{1}{2}(\mu_1 - \mu_2) \end{aligned}$$

Thus $x$ can be calculated accurately however small $\mu_1$ and $\mu_2$ may be. Both parts of the expression for $t$ have therefore been reduced to a form free from difficulty.

14. If the value of $x$ is very small, as it may be for a short arc, an approximation to the value of $\operatorname{Log}(1 + x) - \operatorname{Log}(1 - x)$ can be found thus. We have

$$\operatorname{Log}(1 + x) - \operatorname{Log}(1 - x) = 2\left(x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \ \ldots\right)$$

But

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \ \ldots$$

Hence

$$\operatorname{Log}(1 + x) - \operatorname{Log}(1 - x) - 2 \tan x = \frac{2}{15}x^5 + \frac{8}{45}x^7 + \ \ldots$$

The approximation $2 \tan (x.\ 206265'')$ holds therefore with fair accuracy if $x < 0{\cdot}1$; and from $0{\cdot}1$ onwards the function can be calculated directly under the form

$$\operatorname{Log}(1 + x) - \operatorname{Log}(1 - x) = \operatorname{Log} \tan (45^\circ + \tan^{-1} x)$$

Note that throughout Log has been used to denote natural logarithm, and log to denote common logarithm, so that

$$\operatorname{Log} = 2{\cdot}3025851 \log = [0{\cdot}3622157] \log$$

*University Observatory, Oxford:*
1909 *January* 7.

---

*Note added* 1909 *January* 19.

The approximation given in § 14 is of some importance, and may be useful for other purposes, *e.g.* in calculating the Guder-

mannian function for small values of the argument. It may be well, therefore, to examine it a little more closely. We have

$$\text{Log}\,\{\text{Log}\,(1+x)-\text{Log}\,(1-x)\}=\text{Log}\,2\left(x+\frac{x^3}{3}+\frac{x^5}{5}+\ldots.\right)$$
$$=\text{Log}\,2x+\text{R}$$

where

$$\text{R}=\text{Log}\left(1+\frac{x^2}{3}+\frac{x^4}{5}+\ldots.\right)$$
$$=\frac{1}{3}x^2+\frac{13}{90}x^4+\frac{251}{2835}x^6+\frac{3551}{56700}x^8+\ldots.$$

On the other hand,

$$\text{Log}\tan x=\text{Log}\,x+\frac{1}{3}x^2+\frac{7}{90}x^4+\frac{62}{2835}x^6+\frac{127}{18900}x^8+\ldots.$$

Hence

$$\text{Log}\{\text{Log}\,(1+x)-\text{Log}\,(1-x)\}-\text{Log}\,(2\tan x)$$
$$=\frac{1}{15}x^4+\frac{1}{15}x^6+\frac{317}{5670}x^8+\ldots.$$
$$=x^4/15(1-x^2),\ \text{very nearly.}$$

Changing to common logarithms we have then

$$\log\{\text{Log}\,(1+x)-\text{Log}\,(1-x)\}=\log\,(2\tan x)$$
$$+[8{\cdot}4616930]x^4/(1-x^2)$$

and the following little table gives the correction additive to log $(2\tan x)$ in the seventh decimal place:—

| $x$. | Corr. | $x$. | Corr. | $x$. | Corr. |
|---|---|---|---|---|---|
| 0·04 | 001 | 0·12 | 061 | 0·16 | 195 |
| ·05 | 002 | ·13 | 084 | ·165 | 221 |
| ·06 | 004 | ·135 | 098 | ·17 | 249 |
| ·07 | 007 | ·14 | 113 | ·175 | 280 |
| ·08 | 012 | ·145 | 131 | ·18 | 314 |
| ·09 | 019 | ·15 | 150 | ·185 | 351 |
| ·10 | 029 | ·155 | 171 | ·19 | 391 |
| ·11 | 043 | ·16 | 195 | ·195 | 435 |
| ·12 | 061 | | | ·20 | 483 |

Thus, for example, when $x=0{\cdot}2$, which is the circular measure of $11^\circ\,27'\,32''{\cdot}96$, we have log (Log $1{\cdot}2/0{\cdot}8$) $=9{\cdot}6079536$ and log $(2\tan x)=9{\cdot}6079053$, the difference coinciding exactly with the last entry in the table.

## *A Method for the Reduction of Comet Photographs.*

By H. C. Plummer, M.A.

1. The magnificent results which have lately been obtained in comet photography suggest some reflections on the method by which the plates may be reduced. The method of Bessel,* which was designed for the treatment of Halley's comet in 1835, and has been followed more or less faithfully by all subsequent writers, deals with observed position-angles and distances with respect to the nucleus. If the observations are not made in this form, they must first be reduced to it. Of course position-angles and distances can be very easily deduced from the photographic plate.† But it has frequently been found ‡ that there is an advantage in the use of rectangular coordinates over spherical geometry in dealing with photographs. There is a corresponding change in the method of calculation from the general use of logarithms to the calculating machine which appeals strongly to some computers. It may, therefore, be worth while to investigate the formulæ which seem to be the natural expression of this point of view. Whether the emanations from a comet remain strictly in the plane of the orbit is open to doubt, but provisionally it is necessary to make this assumption, and it forms the basis of what follows.

2. It will be convenient to give precepts for certain preliminary operations. Let us form the arrays

$$\begin{vmatrix} -\sin A & \cos A & 0 \\ -\sin D\cos A & -\sin D\sin A & \cos D \\ \cos D\cos A & \cos D\sin A & \sin D \end{vmatrix} \quad . \quad (1)$$

where A, D are the R.A. and declination of the plate-centre, and

$$\begin{vmatrix} X & Y & Z \\ a\sin(\alpha+u) & b\sin(\beta+u) & c\sin(\gamma+u) \\ a\cos(\alpha+u) & b\cos(\beta+u) & c\cos(\gamma+u) \end{vmatrix} \quad . \quad (2)$$

where X, Y, Z are the rectangular coordinates of the Sun referred to the equatorial system of coordinates, $a$, $b$, $c$, $\alpha$, $\beta$, $\gamma$ the Gaussian constants for the orbit of the comet, and $u$ an angle measured from the ascending node in the direction of the comet's motion: the actual value to be assigned to $u$ can be left for definition later on. Next we form the array

$$\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} \quad . \quad . \quad . \quad . \quad (3)$$

* *A.N.*, No. 300.

† Cf. *M.N.*, lxix. p. 100.

‡ Cf. *M.N.*, lx. pp. 176 and 201; lxii. p. 29.

which is formed by the products by rows from the arrays (1) and (2), so that

$$
\begin{aligned}
l_1 &= -\sin A \,.\, X + \cos A \,.\, Y \\
m_1 &= -\sin A \,.\, a \sin(\alpha+u) + \cos A \,.\, b \sin(\beta+u) \\
n_1 &= -\sin A \,.\, a \cos(\alpha+u) + \cos A \,.\, b \cos(\beta+u) \\
l_2 &= -\sin D \cos A \,.\, X - \sin D \sin A \,.\, Y + \cos D \,.\, Z \\
m_2 &= -\sin D \cos A \,.\, a \sin(\alpha+u) - \sin D \sin A \,.\, b \sin(\beta+u) + \cos D \,.\, c \sin(\gamma+u \\
n_2 &= -\sin D \cos A \,.\, a \cos(\alpha+u) - \sin D \sin A \,.\, b \cos(\beta+u) + \cos D \,.\, c \cos(\gamma+ \\
l_3 &= \cos D \cos A \,.\, X + \cos D \sin A \,.\, Y + \sin D \,.\, Z \\
m_3 &= \cos D \cos A \,.\, a \sin(\alpha+u) + \cos D \sin A \,.\, b \sin(\beta+u) + \sin D \,.\, c \sin(\gamma+u \\
n_3 &= \cos D \cos A \,.\, a \cos(\alpha+u) + \cos D \sin A \,.\, b \cos(\beta+u) + \sin D \,.\, c \cos(\gamma+
\end{aligned}
$$

Lastly we form the conjugate array

$$
\begin{vmatrix} L_1 & M_1 & N_1 \\ L_2 & M_2 & N_2 \\ L_3 & M_3 & N_3 \end{vmatrix} \quad . \quad . \quad . \quad . \quad (4)
$$

the constituents of which are the minors

$$
\begin{aligned}
L_1 &= m_2 n_3 - m_3 n_2, \quad M_1 = l_2 n_3 - l_3 n_2, \quad N_1 = l_2 m_3 - l_3 m_2 \\
L_2 &= m_1 n_3 - m_3 n_1, \quad M_2 = l_1 n_3 - l_3 n_1, \quad N_2 = l_1 m_3 - l_3 m_1 \\
L_3 &= m_1 n_2 - m_2 n_1, \quad M_3 = l_1 n_2 - l_2 n_1, \quad N_3 = l_1 m_2 - l_2 m_1
\end{aligned}
$$

The operations involved in computing the constituents of the arrays (3) and (4) will be very easily carried out with the help of an arithmometer.

3. Let $\rho, \theta$ be the polar coordinates of a point in the plane of the orbit, the origin being the Sun and the initial line making an angle $u$ with the direction of the node. Then the geocentric coordinates of the point are

$$
\begin{aligned}
x_1 &= a\rho \sin(\alpha+u+\theta) + X \\
&= ax \sin(\alpha+u) + ay \cos(\alpha+u) + X \\
y_1 &= bx \sin(\beta+u) + by \cos(\beta+u) + Y \\
z_1 &= cx \sin(\gamma+u) + cy \cos(\gamma+u) + Z
\end{aligned}
$$

But if $x_2, y_2, z_2$ are the coordinates of the same point referred to the axes O A, O B, O C, where C is the plate-centre, and the plane O C B passes through the pole P,

$$
\begin{aligned}
x_2 &= -\sin A \,.\, x_1 + \cos A \,.\, y_1 \\
y_2 &= -\sin D \cos A \,.\, x_1 - \sin D \sin A \,.\, y_1 + \cos D \,.\, z_1 \\
z_2 &= \cos D \cos A \,.\, x_1 + \cos D \sin A \,.\, y_1 + \sin D \,.\, z_1
\end{aligned}
$$

the coefficients being given by the array (1). Hence

$$
\begin{aligned}
x_2 &= l_1 + m_1 x + n_1 y \\
y_2 &= l_2 + m_2 x + n_2 y \\
z_2 &= l_3 + m_3 x + n_3 y
\end{aligned}
$$

But if $\xi, \eta$ are the standard coordinates of the image of the point on the plate, $f$ is the focal length, and parallax is neglected,

$$
\xi / x_2 = \eta / y_2 = f / z_2
$$

so that $x$, $y$, the rectangular coordinates in the plane of the orbit, are connected with $\xi$, $\eta$ by the relations

$$(l_1 + m_1 x + n_1 y)/\xi = (l_2 + m_2 x + n_2 y)/\eta = (l_3 + m_3 x + n_3 y)/f \quad (5)$$

which are equivalent to

$$L_1\xi - L_2\eta + L_3 f = (-M_1\xi + M_2\eta - M_3 f)/x = (N_1\xi - N_2\eta + N_3 f)/y \quad (6)$$

Thus the sets of coefficients in (3) or (4) enable us to pass easily from points in the plane of the orbit to the corresponding points on the photograph, or *vice versa*.

4. The value to be assigned to $u$ has purposely been left arbitrary. If, for example, we put $u = 0$, the axis of $x$ will be the line from the Sun to the ascending node of the comet. But if we wish to form a representation of a number of different points connected with the comet at the date of a particular photograph, it is natural to make the radius vector of the nucleus the axis of $x$, and then we must put

$$u = \varpi - \Omega + v$$

where $v$ is the true anomaly of the nucleus at the date of the photograph. On the other hand, if we wish to follow the actual motion of a particular point as shown by a series of photographs at different times, it will be advantageous to use a fixed axis of $x$ by putting

$$u = \varpi - \Omega + v_0$$

where $v_0$ is a constant chosen for convenience, and may be the true anomaly of the nucleus at the time of one of the photographs, or may be chosen for some particular reason; *e.g.* if we put $v_0 = 0$, the axis of $x$ will be drawn from the Sun in the direction of perihelion. In any case the Sun is at the origin of coordinates.

5. The projective relation between corresponding points on the photograph and on the plane of the comet's orbit make it easy to transform any curved aggregate of points on the photograph into the corresponding curve in space by means of (5). Thus if we observe a straight streamer whose equation on the plate is

$$A\xi + B\eta + C = 0$$

this corresponds in the plane of the orbit to

$$Al_1 + Bl_2 + Cl_3 + (Am_1 + Bm_2 + Cm_3)x + (An_1 + Bn_2 + Cn_3)\eta = 0$$

And similarly any more complicated curve on the plate for which the equation can be found can be transferred to the plane of the orbit with far greater ease than by computing a series of positions point by point.

6. When a particular condensation in the matter emitted by a comet can be observed on a series of plates, it may be possible to determine the motion, and hence to infer the law of force which is acting. The difficulties in the way of identifying such a conden-

sation and measuring it with any approach to accuracy are very great. Hitherto attempts to do this have not met with very great success. But it is a problem of immense interest. If the equation of the orbit is

$$p = \rho\{1 + e\cos(\theta - \gamma)\}$$

and we have a series of positions $(x_r, y_r)$ at times $t_r$, of which the number must be at least three, the constants will be given by the equations

$$p = \rho_r + ex_r\cos\gamma + ey_r\sin\gamma$$

for these will determine $p$, $e\cos\gamma$ and $e\sin\gamma$. When these have been found, three cases may arise according as (1) $e<1$, (2) $e>1$ and $p$ is positive, (3) $e>1$ and $p$ is negative, these corresponding to motion on an ellipse, on the concave and on the convex branch of an hyperbola respectively.

7. It may be convenient to set down here the formulæ which suffice to determine the constant of force in each of the three cases.

(1)
$$\begin{aligned} &x = \rho\cos\theta\,,\ y = \rho\sin\theta\,,\ v = \theta - \gamma \\ &e = \sin\phi\,,\ a = p\sec^2\phi \\ &\tan\tfrac{1}{2}\mathrm{E} = \tan(45^\circ - \tfrac{1}{2}\phi)\tan\tfrac{1}{2}v \\ &\mu^{\frac{1}{2}}a^{-\frac{3}{2}}(t - \mathrm{T}) = \mathrm{E} - e\sin\mathrm{E} \\ &\mu = k^2(1 - \mu') \end{aligned}$$

which determines $\mu'$, the ratio of the repulsive action to gravitation, $k$ being the Gaussian constant.

(2)
$$\begin{aligned} &x = \rho\cos\theta\,,\ y = \rho\sin\theta\,,\ v = \theta - \gamma \\ &e = \sec\phi\,,\ a = p\cot^2\phi \\ &\tan\tfrac{1}{2}\mathrm{F} = \tan\tfrac{1}{2}\phi\tan\tfrac{1}{2}v \\ &\mu^{\frac{1}{2}}a^{-\frac{3}{2}}(t - \mathrm{T}) = e\tan\mathrm{F} - \mathrm{Log}\tan(45^\circ + \tfrac{1}{2}F) \\ &\mu = k^2(1 - \mu') \end{aligned}$$

which determines $\mu'$ as before. Log denotes natural logarithm.

(3) For clearness, when $p$ as determined in § 6 is negative, we simply change its sign and rewrite the equation of the convex branch

$$p = -\rho + e\rho\cos(\theta - \gamma - \pi)$$

and then

$$\begin{aligned} &x = \rho\cos\theta\,,\ y = \rho\sin\theta\,,\ v = \theta - \gamma - \pi \\ &e = \sec\phi\,,\ a = p\cot^2\phi \\ &\tan\tfrac{1}{2}\mathrm{G} = \cot\tfrac{1}{2}\phi\tan\tfrac{1}{2}v \\ &\mu^{\frac{1}{2}}a^{-\frac{3}{2}}(t - \mathrm{T}) = \mathrm{Log}\tan(45^\circ + \tfrac{1}{2}\mathrm{G}) + e\tan\mathrm{G} \\ &\mu = k^2(\mu' - 1) \end{aligned}$$

which again determines $\mu'$ in terms of gravitation. Astronomical units are assumed throughout.

8. If three observations of a condensation are available, it is evident that only a pair of times is required to determine $\mu$ and T, and that the time of the third observation serves as a control. A more satisfactory control must be looked for in a greater number

of observations, and the success with which they can be brought into harmony. If discrepancies present themselves, there are only too many causes which may contribute to produce them. These include not only false identifications and want of accuracy in a particularly difficult class of measurement, but also the possibly false assumption that the matter is moving in the plane of the orbit, and an error in the hypothesis that the repulsive force varies as the inverse square of the distance. It is to be hoped that those who have reaped such a brilliant success in the field of photographic observation will be undaunted by the patent difficulties, and will arrive at some more definite conclusions as to the laws of cometary forms than we can be said to have reached at the present time.

*University Observatory, Oxford:*
1909 *January* 7.

---

*The Clouds of Venus and their Significance.* By Arthur W. Clayden, M.A., Principal of the Royal Albert Memorial University College, Exeter.

If it be assumed that the visible surface of Venus is the outer face of a heavily cloud-laden atmosphere, comparable in mass and composition with our own, it seems that certain deductions may be based on its observed features which have not hitherto been pointed out, though they convey valuable hints as to the planet's physical condition, the position of its axis, and the period of rotation.

There is no need to recapitulate here the many reasons for believing that the surface visible to most telescopes is actually cloud, and the sequel will show that such a view is not incompatible with the detection of sharp markings such as have been described by specially favoured observers. My own observations have been made with the 6·8 inch refractor by Tulley, described in vol. ii. of the Society's *Memoirs*. Its long focus (144 inches) gives excellent definition of Jupiter and Saturn, when the seeing is good, with powers up to 450; but it has never shown me anything on Venus which does not seem easily explicable on the assumption that the planet is shrouded in clouds, and that its atmosphere bears something like the same relation to its mass as our own atmosphere does to the Earth's.

Taking this for granted, let us consider how those clouds should be distributed according to different views as to the period of rotation and the position of the polar axis.

First let us assume that the axis makes a large angle with the plane of the orbit, and that the period of rotation is coincident with the revolution. That is to say, that the planet always turns the same face to the Sun—one side always exposed to the solar glare,

and the other feeling the full effect of continuous radiation into space.

In this case, even allowing for considerable libration, it is more than difficult to imagine how the clouds are maintained on the sunlit face at all. The whole of the water should long ago have been distilled on to the dark side, and there locked up in the form of ice. It seems impossible that there should be an atmospheric circulation capable of keeping up a high enough temperature on the cold hemisphere to allow of the return to the hot one of a sufficiently large amount of water to produce the clouds. We cannot call to our aid a circulation of water in the liquid state, for rivers are dependent upon rainfall or thawing ice, and the very seas should have been distilled. A slow glacial creep seems the only possibility, and that could hardly suffice to do more than produce a ring of cloud along the terminator, leaving the greater part of the hot hemisphere to stand out naked to the sunshine and to our view. Moreover, the phenomena seen along the terminator show no symptoms of great and rapid movements of the atmosphere, such as should certainly be the case if the continuity of a reasonable balance of temperature, and therefore of humidity, were dependent even indirectly upon them.

It may be contended with some reason that the great pall of clouds will moderate the temperature of the sunlit face, but this hemisphere must, on the whole, be one of rising convection currents, while the dark side must conversely be one of descending currents. Descending movements mean clear skies, and therefore unchecked radiation; so, whatever the initial temperature of the dark face may have been when last shut off from the Sun's heat, it cannot have been protected by a blanket of cloud, and must be intensely cold.

The assumption of a high internal temperature is no more satisfactory, for the same causes which should have frozen the water should have previously chilled the solid rocks, and have thereby placed a thick protecting shield between the hot interior and the frozen surface. If we allow for the difference in the internal pressures, the densities of Venus and the Earth are so near as to imply not only identity of composition, but a close similarity of condition.

It seems that the only way in which the persistence of the cloudy envelope could be explained would be to assume a thermal conductivity for the mass of the solid planet far higher than that of the materials of the Earth.

In the absence of any such extravagant and improbable supposition, it thus appears that the continued existence of the cloudy shell affords an almost conclusive negative to the possibility of an exact identity between rotation and revolution.

Next, suppose the axis to make a large angle with the plane of the orbit, and that the rotation period is not very different from the length of a terrestrial day.

In this case it is at once obvious that the distribution of

temperature on the planet should be strictly similar to that on the Earth. The actual values might be everywhere higher, but the gradients should be in precisely the same directions. Of course the local variations due to the distribution of land and water would differ, but the broad differences due to latitude and the time of day should present an exact parallel.

Moreover, the laws of motion which govern the movements of our atmosphere, and combine with the results of the temperature gradients to determine the general arrangement of atmospheric pressure and the great wind systems, must inevitably produce effects on the somewhat smaller planet, just like those which dominate the meteorology of the Earth.

The regions of ascending or cloud-producing currents must be similarly distributed, and the descending or cloud-destroying currents must occupy the same relative places.

The result should be a belt of uprising currents surrounding the planet's equator at its equinoxes, and moving more or less towards each pole in turn with the progress of the seasons. North and south of this, at about the latitude of 20° or 30°, there should be a belt of higher pressure, and therefore of descending motion, while further away from the equator towards each pole the chief drift of the atmosphere should be tangential and moving with the rotation.

There should therefore be a shining white belt over the equatorial low pressure, flanked on either side by a band where clouds should be small and scanty, and therefore much less brilliant. Then again, brighter regions should extend to each pole.

Moreover, such a circulation should give rise to storms analogous to our own, moving over the planet in the same way, and their courses should often be clearly visible.

The Earth must show all these features so plainly that they would be the first things noticed. Except in the region of our tropical belts of high pressure an astronomer on Venus would rarely be able to make out much of terrestrial geography. The belt of clouds marking the position of the equatorial rainy season must stand out as a shining white band, in strong contrast with the darker belts on either side. All detail of the world's temperate zones must be masked in the most annoying way, for it must be very rare that a district so large as the British Isles is entirely free from cloud, even if we exclude cirrus from consideration.

There is no question that the Earth is a belted planet; and if Venus had a rotation period comparable with our day, and if her atmosphere is of similar or greater density than ours (which is essential for the formation of the shell of convection clouds), she also would take a place in the same group. It is true that Mars does not, but its small mass and the low value of gravity are sufficient to account for the absence of the lower clouds, and these are those which form the belts.

There seems, therefore, no reasonable doubt that if the polar axis of Venus forms a large angle with the plane of the orbit, her

rotation period cannot be anything like so short as the length of our own day.

Suppose, next, that the axis is very little inclined to the plane of the orbit, as has been sometimes suggested.

Apart from all mechanical difficulties in the way of such a supposition, the behaviour of the cloud stratum affords a sufficient test.

Each pole will in turn be heated and cooled, so the general distribution of temperature will be different. But at each equinox the temperature gradients should be in directions similar to those presented in the last case. From the equator towards the pole last presented to the Sun the gradient would be small, but it would be very steep towards the other. The planet should therefore show a bright equatorial band shading away towards the warmer pole, and flanked on the side towards the colder pole by a clearly marked dark belt.

That it does not show a trace of any such feature, or any other symptom similar to the former case, is a sufficient proof that the supposition is not a fact.

It thus appears that the short rotation period cannot be true in any position of the axis, and that the planet's day and year cannot be identical. In one case we ought to see things which we do not, and in the other we ought not to see what we do.

The objections to a short period are equally valid for any period shorter than our own day, and for any period longer than the Earth's up to one as long probably as our week or more, but the difficulty with the long period is only applicable to exact identity between the planet's day and year. If the actual length of its day falls only a little short of the time of revolution, all parts of its surface will in turn experience the extremes of heat and cold. Probably any period between twenty of our days, on the one hand, and, say, 180 or 200 days, on the other, would give conditions in which no cloud bands or high-pressure tropical belts would be produced parallel to the equator, and no permanent locking up of the water could occur.

The estimates made, with the exception of Bianchini's 24 days 8 hours, all indicate a period similar to our own day, or one nearing the length of the planet's year. If the markings upon which these measurements were based were actual features of the planet, or effects produced upon the clouds by underlying features, and not mere atmospheric phenomena, owing their position to the time of day or drifting wind, it seems most probable that Venus rotates upon her axis in a time which approaches the length of her year, but differs from it sufficiently to enable water to be carried across the sunlit face without complete evaporation.

Upon this supposition, it seems easy to explain several of the best known and most easily observed phenomena presented by the planet in its different phases, including that error of phase which has not hitherto received any satisfactory interpretation.

Suppose, then, that the axis is highly inclined to the plane of the orbit, and that the rotation period is long — so long that the Law of Equal Areas, Ferrel's Law of the deflection of currents moving over a rotating body, and the laws of centrifugal force do not exercise any dominating effect on the atmospheric circulation.

Under these circumstances, temperature on the sunlit face will be lowest at the poles and along the sunrise terminator. It will increase to a maximum at a point on the equator west of the noon meridian, and will decrease towards sunset and across the dark side until the day is again breaking. The hottest region will not be a belt, but an oval area, with a steep gradient of falling temperature towards the poles and sunrise terminator, and a gentler gradient towards the sunset.

There should be, therefore, a region of expansion over the hottest area, from which the upper layers of the air should flow radially outwards in all directions, following the gradients of decreasing temperature, exactly as our atmosphere overflows from the equatorial low-pressure belt to the tropics.

Just as the terrestrial overflow results in the overloading of the lower air along the two anticyclonic belts of Cancer and Capricorn, so in the case of Venus the radial overflow must create a ring of higher pressure about 30° to 40° away from the maximum temperature, and therefore low pressure, on every side.

In the case of the Earth the bands are irregular, and frequently incomplete, owing to the distribution of land and water. This would probably be the case with Venus. The exact position and intensity of the high-pressure ring should vary slightly from time to time as the diverse hidden features of the planet pass beneath it on their journey across the disc. Nevertheless, it should always be there, and, from the difference in the temperature gradients towards sunrise and sunset, it should be much more marked on the morning side than on the other. In our own atmosphere we have a clear indication of a similar tendency to the formation of a ring of high pressure around the areas of greatest heat in the constant small rise of the barometer between 9 and 10 A.M.

Beyond the high pressure ring there should be a fall of pressure towards the terminator on every side into the darkness, but the pressure should rapidly increase again to a great maximum covering a large part of the dark hemisphere.

Low-pressure regions mean ascending currents and the abundant formation of convection clouds of the cumulus, cumulo-nimbus, and cyclonic types. High-pressure regions, on the other hand, mean descending movements and scanty formation of convection clouds. Even when the sunshine does produce local ascending currents in such a region, they are stunted by having to rise against the general descent, and the cumulus clouds produced are small and scattered, leaving wide interspaces through which descent goes on.

The whole area on the face of Venus within the high-pressure

ring should be covered with ascending cloud columns; and if moisture is abundant enough, a dense mass of convection clouds, like those which accompany our equatorial rains, should reach high up above the body of the planet.

The high-pressure ring should be chequered with smaller detached cumulus, like those of our trade wind zones, less brilliant because deeper down, and because the interspaces should let us see still deeper, even perhaps to the shaded surface of the planet itself. Here, if anywhere, there would be the best chance for us to glimpse something of the planetary geography, but any markings would look as if seen through a luminous mist caused by the detached clouds.

Beyond this ring of lesser brightness the convection clouds should again rise high above the surface, but should rapidly lessen in altitude as the terminator is approached.

At eastern elongation the terminator is probably sunrise and the clouds are growing. As they are carried slowly forward into the day they should rise higher and higher. On reaching the high-pressure ring they would be degraded, broken, and perhaps to a great extent destroyed. But beyond this they would be reinforced by still stronger convection until the hottest region had been crossed. From this point degradation should again set in, and towards sunset there would be a rapid fall in their level, so that their upper surfaces should slope rapidly downward into the night.

Now, convection currents rising from the ground do not lift their burden of cloud particles indefinitely. The very formation of the cloud, by loading the air with particles of liquid water or solid ice, tends to check the ascent, and the thermal effects of expansion, with the fall of rain and hail combine to put a limit to their possible stature. On the Earth they do not reach beyond 5 or 6 miles from the ground.

But the Sun's rays falling on the upper surface of a stratum of such clouds evaporates water from them, and this is carried upward by the general ascent of the air over a low-pressure area until it reaches such altitudes as 9 or 10 miles, or more. Here it condenses into fine particles of ice, and forms the great variety of translucent clouds classed as cirrus and cirro-nebula.

When we bear in mind the greater intensity of solar radiation on Venus, we see that both convection from the ground and this upper circulation should take place on a grander scale than anything we experience. Its atmosphere should probably be richer in water vapour, both classes of cloud should be more abundant, and, owing to the smaller value of gravity, the altitudes reached should be greater.

Since atmospheric pressure should, on the whole, be considerably greater on the dark side than on the one turned towards the Sun, the upper currents should flow from the hot side over the high-pressure ring and towards the terminator, while colder under currents should flow in the opposite direction.

The cirrus clouds floating in the upper currents should thus pass constantly across the terminator on all sides, and persist some

distance beyond. They are always thin, and generally scattered. If the base of the atmosphere on Venus is at a higher temperature than is the case with the Earth, a supposition which is almost certainly true of the greater part of the sunlit face, cirrus clouds should not form until an altitude is reached at which the pressure is lower than that required for their formation here. They should, therefore, be loftier and even thinner than ours, and ours do not hide the Sun. They are shining white by transmitted light, and shine most brilliantly when viewed edgewise.

The whole sunlit face of Venus should thus be covered with a delicate filmy network of cirrus, similar to that which enfolds the Earth, varying in density, but transparent as a whole when viewed at a considerable angle, and becoming whiter and more opaque as the obliquity of the point of view increases.

Now there are certain distinct phenomena which have been repeatedly described, and which may be seen under suitable circumstances with quite moderate telescopes, which seem to me to admit of simple explanation if the foregoing hypotheses are correct.

1. The brightening of the limb relative to the rest of the disc.

This is a distinctive feature of all planetary bodies which may be reasonably supposed to have nothing but at most a thin, pure atmosphere above the reflecting surface.

In the case of Venus it may be explained as due in the first instance to the fragmentary veil of cirrus, which stands out more and more brilliant as it is seen more obliquely. Being higher in the atmosphere, its particles would be brighter, area for area, than those of the lower clouds, whose summits would be seen through a greater thickness of air, and would also to some extent be shaded by the cirrus itself.

If the cirrus sphere could exist in space alone, we should see only a slender ring of light, very lustrous on its external margin, but rapidly fading away to a dark interior.

2. The prolongation of the cusps.

If the cirrus veil is, as would almost certainly be the case, some miles above the convection clouds, it would catch the light several degrees beyond the true terminator. Since the air at such an altitude is practically free from dust, and since all the lower layers of the atmosphere are densely laden with opaque cloud, we cannot expect that even the tips of the horns should be illuminated by sunset colouring. This can only be produced when the rays have passed through the lower air. The light should therefore be sensibly white, or at most a pale yellow. The thinning of the cusp to a fine, sharp line would be due to the fact that as the intensity of illumination diminishes with distance from the true terminator, the film of cirrus will need to be seen more and more obliquely for a given brightness. It should, therefore, thin out exactly as it does.

3. Detached points beyond the tips of the horns, and occasional absence of the horns.

Cirrus clouds are not continuous. They do not form a con-

tinuous layer around the Earth, nor is there any reason to suppose they should do so on Venus. Hence, if one of the clear spaces should happen to be in the right position, we should see through the gap, and the horn would be absent or broken.

Moreover it may be assumed that the planet is diversified by mountains and valleys like our own, and that these are probably of similar stature and depth.

When the great air-currents flow over such features, up and down movements are produced, which react upon any cloud zones above, destroying the clouds by a descent in one place, creating them or throwing them up to an abnormal height by an ascent elsewhere.

We can thus explain irregular or even periodic appearances of projections of the bright surface, or the occurrence of dark spaces in it, by attributing them to the passage of the cloud-laden air over an uneven surface beneath. Projecting points of light may well be thrown up to very great altitudes without needing to suppose mountains so vast that they would flatten out under their own weight.

4. The acceleration of phase in eastern elongation is commonly observed to vary from 4 to 8 days,—dichotomy, or any special phase, being so much earlier than the theoretical time.

This can only mean that the surface along the sunrise terminator is not at right angles to the planet's radius, but that it slopes upwards towards the Sun, and that the slope is steepest in the equatorial regions, and diminishes towards the poles.

But this is exactly how the upper surface of a shell of rising convection clouds should rise, and the fact alone is an almost conclusive proof that the cloud theory is right.

The phenomenon is not limited to dichotomy, but can be noticed at any time not far removed from greatest elongation.

The variation of the acceleration would imply a variation in the slope such as would be a natural consequence of changes in the underlying surface due to the slow rotation relative to the Sun which has been postulated as essential to the permanent maintenance of the clouds.

5. The retardation of phase in western elongation is rather more pronounced. It can be explained as due to an even greater slope of the mean surface of the clouds, due to the collapse of the rising currents and breaking down of the clouds towards sunset. On the Earth the fall of the cloud levels in the evening is distinctly more rapid than their morning rise. The fact may be regarded as a proof confirmatory of the last.

6. The shading off of the terminator and,

7. The variable contour of the terminator would both be the natural result of variations in the height to which the clouds had reached in consequence of differences in the nature, and therefore temperature or moisture of the surface beneath.

8. The smaller luminosity of the neighbourhood of the terminator..

Apart from the glittering margin, which has been attributed to the cirrus veil, and apart from any definite darkening of a particular region, this is perhaps the most obvious feature of the planet.

At the limb we see only the shining cirrus. Elsewhere we look through it on to the summits of the convection clouds. If, as appears to be almost certain, the individual clouds have on the whole a roughly pyramidal or pillared contour like our cumulus and alto-cumulus castellatus, the body of the planet would be brightest near the limb, where only the tips of the clouds would be seen. On passing from the limb towards the centre of the disc, we should see further and further into the shadows between them, and the proportion of shade should increase as the Sun's rays become more oblique.

9. The presence of a faint band of shade, approximately parallel to, but at some distance from, the terminator.

This can nearly always be seen. It is not sharply contoured, but broad and ill-defined. Still a graduated shade, such as a neutral tint wedge, will generally bring it out. It is best seen in eastern elongation when the planet is near dichotomy, or approaching its maximum brilliancy. It varies from time to time in breadth and depth of shade, but always occupies the same position relative to the terminator. It cannot apparently be followed into the shining ring near the limb.

Surely this is the high-pressure ring, darker than the rest because we see deeper into it, and the spaces between the clouds are larger. It is much less pronounced in western elongation, as we should expect should be the case from the difference between the temperature gradients towards sunrise and sunset.

10. The mottling or granulation of the surface, the appearance of dark or bright spots moving over the terminator, and the occurrence of bright points beyond it, are all of them things which should be visible, and receive obvious explanation.

11. The ring of light seen at inferior conjunction, and usually attributed to atmospheric refraction, would more probably be nothing but the edge of the illuminated shell of cirrus. This stratum would everywhere around the terminator be several miles above the lower clouds, and perhaps 10 or 20 miles above the actual surface of the planet. Mere refraction ought to give a ring of coloured light, yellow, or even red. But if the lower air is, as has been supposed, blocked by heavy convection clouds, all coloured rays would be stopped, while the high cirrus would shine with the silvery light we see.

At such altitudes atmospheric refraction would be almost negligible, and a calculation based upon the position in which the circle is complete would give the height at which these delicate structures float, if only we knew how much to allow for that rise of the lower clouds near the terminator to which we have attributed the error of phase.

The phenomenon, then, may be regarded as only another aspect

of the same fact which also explains the prolongation of the cusps.

Finally, if we suppose that the convection clouds do not form a continuous sheet, but are distributed broadcast, with clear intervals for the descending partial currents which should accompany them, it becomes easy to believe that, under suitable circumstances and with sufficiently powerful optical means, it may sometimes be possible to penetrate the veil, and see at least some of the most salient features of the planet's actual surface. On the other hand, the underlying surface must react upon the clouds above, and it may well be that those sharply defined markings which have been recorded are nothing more than a result marked out upon the veil by the features hidden beneath.

---

*The Eclipse of Hipparchus.* By J. K. Fotheringham.

(*Communicated by* P. H. Cowell, M.A., F.R.S.)

In Cleomedes, *De Motu Circulari Corporum Cœlestium*, ii. 3, we find the passage:

"Ολος ποτὲ ἐν Ἑλλησπόντῳ ἐκλείπων ἐτηρήθη ἐν Ἀλεξανδρείᾳ παρὰ τὸ πέμπτον τῆς ἰδίας ἐκλείπων διαμέτρου, ὅπερ ἐστὶ κατὰ τὴν φαντασίαν παρὰ δακτύλους δύο καὶ βραχύ.

The eclipse to which reference is here made is also mentioned by Pappus * in the following words:

ἐν γὰρ τῷ πρώτῳ περὶ μεγεθῶν καὶ ἀποστημάτων λαμβάνει [sc. Ἵππαρχος] φαινόμενον τοῦτο· ἔκλειψιν ἡλίου ἐν μὲν τοῖς περὶ τὸν Ἑλλήσποντον τόποις ὅλου τοῦ ἡλίου ἀκριβῶς γεγενημένην ὥστε μηδὲν αὐτοῦ παραφαίνεσθαι, ἐν Ἀλεξανδρείᾳ δὲ τῇ κατ' Αἴγυπτον τὰ δ μάλιστα πεμπτημόρια τῆς διαμέτρου ἐκλελοιπότα.

The latter passage is to be found, copied verbatim, in Theon.†

It is clear from these two passages that Hipparchus recorded a solar eclipse which was total at or near the Hellespont and was about four-fifths total at Alexandria. It is also clear from the context that Hipparchus used this eclipse in order to obtain the lunar parallax with a view to determining the distances and magnitudes of the Sun and Moon, and, as Dr. Hultsch observes, the values obtained were, like his value for precession, superior to the values afterwards obtained by Ptolemy, and were not surpassed by any modern astronomer before Halley.‡ In Cleomedes the computation is based upon the known distances of Alexandria and the Hellespont from Rhodes; as Rhodes was the home of Hipparchus, there is every reason to believe that these distances

* Quoted, with full *apparatus criticus*, by Hultsch, *Hipparchos über die Grösse und Entfernung der Sonne, Berichte der sächsischen Gesellschaft*, lii. (1900), *Phil.-hist. Classe*, pp. 169–200.

† *Ibid.* ‡ *Ibid.*

are derived from him, and that, in spite of the vagueness of the phrase ἐν τοῖς περὶ τὸν Ἑλλήσποντον τόποις, Hipparchus considered himself justified in treating the eclipse as total at the Hellespont itself. As Dr. Hultsch points out, the phrase ἐν τοῖς περὶ τὸν Ἑλλήσποντον τόποις is also to be found in Hipparchus' extant commentary on Aratus, where a particular duration for the longest day is said to hold good for that locality. This phrase in the passage quoted from Pappus is therefore in all probability derived from Hipparchus, and it would also appear that Hipparchus had obtained other information besides the record of the eclipse from the district of the Hellespont. The Hellespont, Rhodes, and Alexandria were regarded both by Hipparchus and by Ptolemy in the Almagest as situated on the same meridian, and it is therefore not improbable that Hipparchus had arranged to have observations made simultaneously at the three places, though, for the purpose of determining the lunar parallax, he only used the two extremes. The plural τοῖς τόποις may imply that Hipparchus had information from more than one place on the Hellespont.

This record, derived as it is from one of the greatest of ancient astronomers, giving the definite magnitude of a solar eclipse for two distinct places, has naturally received considerable attention. A full discussion of the eclipse, containing references to the work of previous astronomers, was contributed by Professor Celoria to the *Memorie della Classe di scienze fisiche, matematiche e naturali*, vol. vii., *Accademia dei Lincei, Seduta del 7 marzo 1880, Serie 3.* Professor Celoria, after an investigation of all possible eclipses from the time of Hipparchus backwards, identified this eclipse with that of Agathocles, −309. Dr. Hultsch* afterwards proposed to identify it with the eclipse of −128 November 20, an eclipse which Professor Celoria had discussed, but had rejected because, when computed with Hansen's values, this eclipse was not total anywhere near the Hellespont. Since then Professor Newcomb has discussed the eclipse in *Monthly Notices*, lxv. (1904), pp. 181–3, and Mr. Lynn in the *Journal of the British Astronomical Association*, xv., 1904–5, pp. 162–4. Professor Newcomb accepts Professor Celoria's identification, while Mr. Lynn, who has apparently overlooked the reference to Hipparchus, proposes the eclipse of +29. It is rather curious that the eclipse of Hipparchus is nowhere mentioned in Professor Ginzel's *Spezieller Kanon der Finsternisse.*

I have little doubt that Dr. Hultsch's identification is correct. The date that he has selected falls in the midst of the astronomical activity of Hipparchus, and it is much easier to believe that Hipparchus compared observations made in his own time than that he happened to find observations made independently on his own meridian at an earlier date, and used them in default of observations of his own. Dr. Hultsch has not attempted to apply the eclipse to the correction of the lunar theory, and it is perhaps for this

* *Loc. cit.*

reason that his article has not received the attention that it deserves.

I have computed the eclipse of −128 November 20 with the following values:—

A. A set of lunar values, communicated to me by Professor Newcomb, embodying the result of a recent study of lunar observations from the Ptolemaic eclipses to the year 1908. In these values the secular sidereal acceleration of the Moon is fixed at 8″·012 a century, and no secular sidereal acceleration of the Sun is admitted.

B. A set of values in which the secular acceleration of the Moon is fixed 1″ higher than in Professor Newcomb's values, the other terms being identical with those in A.

C. A set of values in which the secular sidereal acceleration of the Sun is fixed at 1″, the other terms being identical with those in A.

D. Mr. Cowell's values, as given in *Monthly Notices*, lxvi. p. 525.

With these values I obtain, by means of Oppolzer's *Syzygientafeln*, the following elements of the eclipse:—

| | Julian Day | G.M.T. h m | L′. ° | Z. ° | ϵ. ° | P. ° | Q. ° | log *p*. | log Δ L. | log *q*. | *w′a*. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A. | 1674630 | 1 19·5 | 236·074 | −2·79 | 23·713 | 9·405 | 11·399 | 0·7001 | 9·7518 | 8·7498 | 0·5459 |
| B. | ,, | 1 7·5 | 236·066 | ,, | ,, | 9·397 | 11·391 | ,, | ,, | ,, | ,, |
| C. | ,, | 1 31·2 | 236·193 | −2·78 | ,, | 9·522 | 11·513 | ,, | ,, | ,, | 0·5458 |
| D. | ,, | 1 31·7 | 236·541 | −2·75 | ,, | 9·898 | 11·877 | 0·6999 | 9·7520 | 8·7501 | 0·5457 |

| | log *fa*. | *μ*. ° | *γ*. | log *n*. | G. ° | K. ° | sin *g*. | sin *k*. | cos *g*. | cos *k*. | sin *δ′*. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A. | 7·6774 | 20·56 | +0·8153 | 9·7539 | 66·64 | 87·24 | $9_n5559$ | 9·9960 | 9·9698 | $9_n1284$ | $9_n5233$ |
| B. | ,, | 17·57 | +0·8147 | ,, | ,, | ,, | $9_n5560$ | ,, | ,, | $9_n1285$ | ,, |
| C. | ,, | 23·44 | +0·8252 | ,, | 66·78 | 87·26 | $9_n5562$ | 9·9961 | 9·9699 | $9_n1263$ | $9_n5240$ |
| D. | 7·6775 | 23·47 | +0·8570 | 9·7541 | 67·07 | 87·27 | $9_n5571$ | ,, | 9·9697 | $9_n1217$ | $9_n5257$ |

| | cos *δ′*. | N′. |
|---|---|---|
| A. | 9·9743 | 98°·2 |
| B. | ,, | ,, |
| C. | ,, | ,, |
| D. | 9·9741 | 98°·1 |

From these elements I deduce the following magnitudes. The computation has in each case been made by Oppolzer's formula as given in his *Canon der Finsternisse*. In the case of Professor Newcomb's values I have made the computation for the middle of the Hellespont both by Oppolzer's formula and by Hansen's

formula as given in Oppolzer's *Canon der Finsternisse.* The results obtained by the two formulæ are identical:—

| | West end of Hellespont. | Middle of Hellespont. | East end of Hellespont. | Alexandria. |
|---|---|---|---|---|
| | ° ′ | ° ′ | ° ′ | ° ′ |
| Latitude | 40 1N. | 40 8N. | 40 22N. | 31 11N. |
| Longitude | 26 9E. | 26 22E. | 26 30E. | 29 51E. |
| Magnitude:— | | | | |
| A. | 11·99 | 11·99 | 12·03 | 9·40 |
| B. | Not computed. | Not computed. | Not computed. | 9·70 |
| C. | ,, | ,, | ,, | 8·87 |
| D. | ,, | 10·81 | ,, | 8·17 |

I have also computed the belt of totality as follows, by means of the formula given in Oppolzer's *Syzygientafeln* [53].

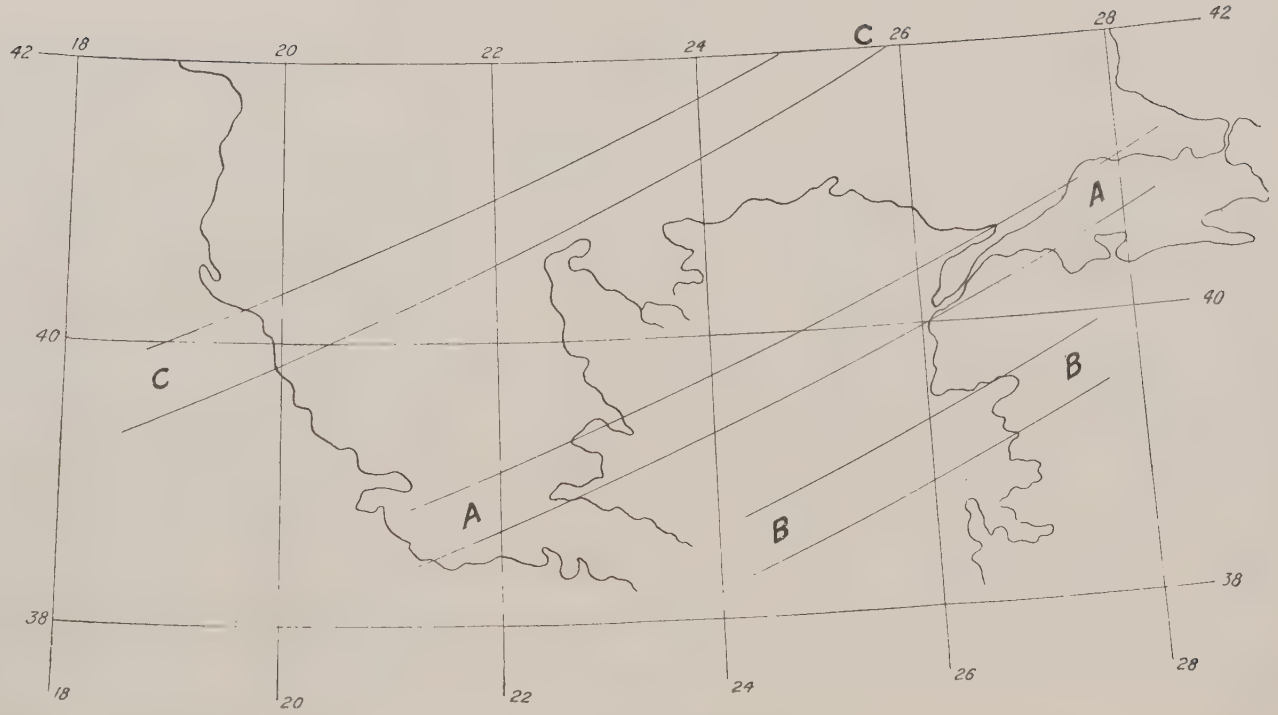
| | | Northern limit. | | | Southern limit. | | |
|---|---|---|---|---|---|---|---|
| | | $t=56°$ | $t=60°$ | $t=64°$ | $t=56°$ | $t=60°$ | $t=64°$ |
| | | ° | ° | ° | ° | ° | ° |
| A. | Latitude | 38·82 | 39·98 | 41·24 | 38·31 | 39·51 | 40·83 |
| | Longitude | 21·28 | 24·81 | 28·47 | 21·14 | 24·67 | 28·35 |
| B. | Latitude | 38·74 | 39·90 | 41·18 | 38·25 | 39·45 | 40·77 |
| | Longitude | 24·25 | 27·78 | 31·45 | 24·12 | 27·66 | 31·32 |
| C. | Latitude | 39·92 | 41·09 | 42·38 | 39·38 | 40·62 | 41·95 |
| | Longitude | 18·70 | 22·24 | 25·94 | 18·54 | 22·10 | 25·80 |
| D. | | Not computed. | | | | | |

The accompanying map shows the belts of totality thus computed (p. 208).

It will be observed that the belt of totality is very narrow, measuring about 25 miles in width. This greatly lessens the range of values that will satisfy the record.

It will also be observed that Professor Newcomb's values make the belt of totality pass along the whole length of the Hellespont, while a very small alteration in the constants would remove the eclipse from the Hellespont altogether. As both sides of the Hellespont were studded with Greek cities, it is probable that the place or places of observation were in the immediate vicinity of that strait. Professor Newcomb's values also give a sufficiently accurate magnitude at Alexandria, 9·40 being very much nearer to four-fifths than to three-fourths, and being readily described as four-fifths. It will be observed that a small increase in the secular acceleration of the Moon would still satisfy the recorded magnitude at Alexandria.

*Eclipse of Hipparchus.* – 128 *November* 20.



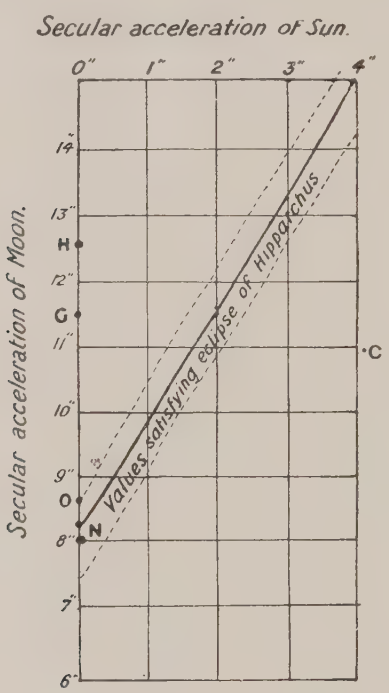

A A, zone of total eclipse computed with Newcomb's values.
B B, ,, ,, ,, secular acceleration of Moon 1″ in excess of Newcomb's value.
1″ secular acceleration of Sun ; other terms from Newcomb.

The centre of Belt C is approximately 123 miles distant from the centre of Belt A, while the centre of Belt A is approximately 68 miles distant from the centre of Belt B. The distances of the belts of totality from each other are therefore in exact proportion to the differences in the magnitudes at Alexandria. As the addition of 1″ to the secular acceleration of the Moon increases D (elongation of the Moon), but does not affect D–F (elongation of the node), while the addition of 1″ to the secular acceleration of the Sun involves a corresponding diminution in D and an equal diminution in D–F, 55 miles out of the 123 between Belts A and C may be attributed to the alteration in D–F, the remainder being due to the alteration in D.

Belt B is at its nearest about 40 miles and Belt C is at its nearest 115 miles distant from the Hellespont. It follows, therefore, that a correction of about $+0''\cdot4$ or of $-0''\cdot1$ to Professor Newcomb's value for the secular acceleration of the Moon is the utmost that it is possible to apply without moving the belt of totality entirely away from the Hellespont. Similarly, $+0''\cdot1$ or $-0''\cdot2$ should be the greatest possible correction to the secular acceleration of the Sun. A further addition of $\pm n''$ to the secular acceleration of the Moon will still satisfy the eclipse, provided that it is counterbalanced by an addition of $\pm\frac{4}{7}n''$ to the secular acceleration of the Sun, where $n$ is any quantity. By placing a liberal interpretation on the phrase *ἐν τοῖς περὶ τὸν Ἑλλήσποντον τόποις* so as to include all places within 20 miles of the Hellespont, we might increase or diminish the secular acceleration of the Moon by $0''\cdot3$, or that of the Sun by $0''\cdot2$. The effect of a correction in the Moon's perigee would be very slight compared with these corrections in the longitude of the Sun and Moon. An addition to the secular acceleration of the Moon's node, generally believed to be impossible, would increase D–F, but would not affect D; the effect of such an addition would be about three-fourths the effect of an equal addition to the secular acceleration of the Moon. It would appear, altogether, that if we do not revise the centennial motions, the eclipse can only be satisfied by a secular acceleration of the Moon amounting to $8''\cdot15\pm0''\cdot55+n''$, accompanied by a secular acceleration of the Sun amounting to $\frac{4}{7}n''$.

Any correction in the centennial motions would involve a corresponding correction of one-nineteenth as much in order to satisfy the record of the eclipse. It is not likely that the resultant correction of the secular acceleration of the Moon would exceed $\pm0''\cdot5$. Professor Newcomb's centennial motion of the Moon at the epoch 1800·0 exceeds Mr. Cowell's by 5″, and the adoption of Mr. Cowell's value would involve a diminution of $0''\cdot25$ in the secular acceleration of the Moon. Similarly, Mr. Cowell's value for the centennial motion of the node exceeds Professor Newcomb's by 8″; the adoption of this value would have to be counterbalanced by the addition of about $0''\cdot3$ to the secular acceleration of the Moon. The combined effect of these two corrections is practically nil.

The following diagram exhibits the different values which will satisfy this eclipse :—




C represents Mr. Cowell's values; G, Professor Ginzel's; H, Hansen's; N, Professor Newcomb's; and O, Oppolzer's.

The eclipse, therefore, affords a striking confirmation of Professor Newcomb's values, but is inconsistent with Professor Ginzel's and Mr. Cowell's. It remains to be seen whether this eclipse can be reconciled with those on which Mr. Cowell's and Professor Ginzel's corrections are respectively founded.

12 *Holywell, Oxford:*
1909 *January* 6.

*Observations made during the partial Eclipse of the Sun on June 28, 1908, at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

| Phenomenon. | Telescope. | Power. | Mean Solar Time of Observation. | Observer. |
|---|---|---|---|---|
| | | | h m s | |
| (*a*) First contact | Dollond (55 Ulundi Road, Westcombe Park) | ... | 5 13 36 | A. C. |
| ,, ,, | Merz Refractor | 250 | 5 13 43·29 | C. D |
| ,, ,, | Great Equatorial (Corbett) | 120 | 5 13 59·7 | W. B. |
| (*b*) ,, ,, | Old Altazimuth | 100 | 5 13 55·89 | H. F. |
| ,, ,, | Astrographic Equatorial | 225 | 5 13 51·36 | P. M. |
| ,, ,, | Sheepshanks Equatorial | 100 | 5 13 45·29 | J. E. |
| (*a*) Last contact | Dollond | ... | 6 1 42 | A. C. |
| ,, ,, | Great Equatorial | 670 | 6 1 41·08 | B. |
| ,, ,, | Merz Refractor | 250 | 6 1 49·33 | C. D. |
| ,, ,, | Great Equatorial (Corbett) | 120 | 6 1 49·46 | W. B. |
| ,, ,, | Old Altazimuth | 100 | 6 1 52·27 | H. F. |
| ,, ,, | Astrographic Equatorial | 225 | 6 1 34·53 | P. M. |
| ,, ,, | Sheepshanks Equatorial | 100 | 6 1 52·37 | J. E. |

(*a*) Observations reduced to Royal Observatory, Greenwich, the corrections being $-1^s{\cdot}38$ and $+2^s{\cdot}03$ respectively; the result is given to the nearest second.

(*b*) Observation doubtful. Probably 5 seconds or more late.

The apertures of the telescopes used are as follows:—

| | |
|---|---|
| Great Equatorial | 28 inches |
| Merz Refractor | $12\frac{3}{4}$ ,, |
| Astrographic Equatorial (guiding telescope) | 10 ,, |
| Sheepshanks Equatorial | $6\frac{3}{4}$ ,, |
| Great Equatorial (Corbett telescope) | $6\frac{1}{2}$ ,, |
| Old Altazimuth | 4 ,, |
| Dollond | 3 ,, |

The initials A. C., B., C. D., W. B., H. F., P. M., J. E., are those of Mr. Crommelin, Mr. Bryant, Mr. Davidson, Mr. Bowyer, Mr. Furner, Mr. Melotte, and Mr. J. Evans, respectively.

## *Observations of Planet* 1908 *DT, from Photographs taken with the 30-inch Reflector at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

This planet, which is of 14–15 magnitude, was discovered on photographs of the region about Saturn taken for observation of Phœbe. It has been measured on 16 photographs taken on 9 nights, and reduced in the same manner as Phœbe.

| Date and G.M.T. | | | | | Apparent R.A. | | | Apparent Dec. | | | Log Parallax Factor. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | R.A. | Dec. |
| 1908. | d | h | m | s | h | m | s | ° | ′ | ″ | | |
| Aug. | 24 | 12 | 20 | 29 | 0 | 40 | 18·04 | +0 | 43 | 53·6 | −9·283 | +0·829 |
| | 25 | 12 | 18 | 51 | 0 | 39 | 56·98 | 0 | 43 | 43·8 | −9·275 | +0·829 |
| | 25 | 14 | 1 | 10 | 0 | 39 | 55·50 | 0 | 43 | 43·1 | −8·529 | +0·828 |
| | 27 | 14 | 8 | 27 | 0 | 39 | 6·00 | 0 | 42 | 56·8 | −7·950 | +0·828 |
| | 28 | 12 | 10 | 16 (*a*) | 0 | 38 | 40·90 | 0 | 42 | 17·3 | −9·263 | +0·829 |
| | 28 | 14 | 3 | 14 | 0 | 38 | 33·84 | 0 | 42 | 19·6 | −8·028 | +0·829 |
| | 29 | 12 | 27 | 47 | 0 | 38 | 10·31 | 0 | 41 | 32·2 | −9·183 | +0·829 |
| | 29 | 13 | 56 | 31 | 0 | 38 | 8·43 | 0 | 41 | 27·7 | −9·171 | +0·829 |
| | 30 | 12 | 3 | 27 | 0 | 37 | 39·06 | 0 | 40 | 30·9 | −9·258 | +0·830 |
| | 30 | 13 | 55 | 31 | 0 | 37 | 36·46 | 0 | 40 | 27·7 | −7·988 | +0·829 |
| Sept. | 1 | 12 | 7 | 33 | 0 | 36 | 29·08 | 0 | 38 | 12·2 | −9·214 | +0·830 |
| | 2 | 12 | 6 | 2 | 0 | 35 | 51·61 | 0 | 36 | 42·1 | −9·203 | +0·830 |
| | 2 | 13 | 29 | 57 | 0 | 35 | 48·55 | 0 | 36 | 36·4 | −8·489 | +0·829 |
| | 2 | 14 | 35 | 31 | 0 | 35 | 47·09 | 0 | 36 | 31·8 | +8·868 | +0·829 |
| | 4 | 11 | 47 | 49 | 0 | 34 | 29·96 | 0 | 33 | 31·0 | −9·240 | +0·831 |
| | 4 | 13 | 11 | 38 | 0 | 34 | 27·01 | 0 | 33 | 25·1 | −8·668 | +0·830 |

(*a*) Very faint and diffused.

*Royal Observatory, Greenwich:*
1909 *January* 8.

*Observations of Saturn's Ninth Satellite,* Phœbe, *from Photographs taken with the 30-inch Reflector at the Royal Observatory, Greenwich, in* 1908.

(*Communicated by the Astronomer Royal.*)

Photographs of Phœbe have been taken with the 30-inch reflector during the recent opposition of Saturn, starting as early as practicable, so as to secure positions at eastern elongation. In all 23 photographs have been secured on 15 nights, between July 31, just before eastern elongation, and September 7, when the series was interrupted on account of Comet *c* 1908 (Morehouse).

The position of the satellite has been measured on photographs taken with the reflector with reference to faint comparison stars (of eleventh or twelfth magnitude), symmetrically distributed about the satellite. The positions of these faint comparison stars were then measured relatively to the reference stars (of eighth to ninth magnitude) on a photograph (with two exposures of $40^m$) taken with the Astrographic 13-inch refractor. The field, sensibly free from distortion, being much larger with this telescope than with the reflector, 32 reference stars were available on the plate. As Saturn moved slowly, it was possible to make one reference plate serve for all the photographs, which were each referred to it. The constants were determined in the usual manner, all the stars on the plate given in the Astronomische Gesellschaft catalogues being used for this purpose. Right Ascensions and Declinations of the satellite were then determined, and compared with the corrected tabular places of Saturn after applying the corrections

in R.A. $+^s\cdot08$ and Dec. $0''\cdot0$

as determined below.

The G.M.T. is the arithmetical mean of the times of beginning and end of exposure.

*Observations of Phœbe.*

| Date and G.M.T. | | | Exp. | Apparent R.A. | | | Apparent Dec. | | | Phœbe − Saturn. R.A. | | Phœbe − Saturn. Dec. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1908. d | h | m | min. | h | m | s | ° | ′ | ″ | m | s | ′ | ″ |
| July 31 | 13 | 27·7 | 90 | 0 | 43 | 23·79 | +1 | 59 | 42·4 | +2 | 19·02 | +17 | 41·0 |
| Aug. 1 | 13 | 35·5 | 90 | 0 | 43 | 20·48 | 1 | 59 | 2·9 | 2 | 19·07 | 17 | 40·6 |
| 2 | 13 | 19·9 | 90 | 0 | 43 | 16·88 | 1 | 58 | 23·1 | 2 | 19·14 | 17 | 41·6 |
| 3 | 13 | 20·6 | 100 | 0 | 43 | 12·94 | 1 | 57 | 40·7 | 2 | 19·28 | 17 | 42·6 |
| 24 | 12 | 20·5 | 100 | 0 | 40 | 26·36 | 1 | 34 | 20·1 | 2 | 18·24 | 17 | 29·8 |
| 25 | 12 | 18·9 | 90 | 0 | 40 | 14·75 | 1 | 32 | 52·3 | 2 | 18·07 | 17 | 28·7 |
| 25 | 14 | 1·2 | 100 | 0 | 40 | 13·83 | 1 | 32 | 45·0 | 2 | 17·97 | 17 | 27·7 |
| 27 | 11 | 54·8 | 70 | 0 | 39 | 50·81 | 1 | 29 | 53·2 | 2 | 17·67 | 17 | 26·4 |
| 27 | 14 | 8·5 | 85 | 0 | 39 | 49·66 | 1 | 29 | 42·8 | 2 | 17·64 | 17 | 24·4 |
| 28 | 12 | 10·3 | 86 | 0 | 39 | 38·13 | 1 | 28 | 17·0 | 2 | 17·41 | 17 | 22·7 |
| 28 | 14 | 3·2 | 90 | 0 | 39 | 37·16 | 1 | 28 | 9·4 | 2 | 17·42 | 17 | 22·4 |
| 29 | 12 | 27·8 | 90 | 0 | 39 | 25·12 | 1 | 26 | 40·5 | 2 | 17·12 | 17 | 20·4 |
| 29 | 13 | 56·5 | 70 | 0 | 39 | 24·33 | 1 | 26 | 36·7 | 2 | 17·11 | 17 | 22·4 |
| 30 | 12 | 3·5 | 90 | 0 | 39 | 12·29 | 1 | 25 | 6·0 | 2 | 16·90 | 17 | 18·7 |
| 30 | 13 | 55·5 | 110 | 0 | 39 | 11·27 | 1 | 24 | 59·7 | 2 | 16·89 | 17 | 19·7 |

*Observations of Phœbe*—continued.

| Date and G.M.T. | Exp. | Apparent R.A. | Apparent Dec. | Phœbe–Saturn. R.A. | Phœbe–Saturn. Dec. |
|---|---|---|---|---|---|
| 1908. d h m | min. | h m s | ° ′ ″ | m s | ′ ″ |
| Sept. 1 12 7·6 | 26 | 0 38 45·43 | 1 21 48·9 | 2 16·49 | 17 14·7 |
| 2 12 6·0 | 80 | 0 38 31·44 | 1 20 10·8 | 2 16·06 | 17 14·9 |
| 2 13 29·9 | 30 | 0 38 30·66 | 1 20 3·0 | 2 16·08 | 17 12·9 |
| 2 14 35·5 | 90 | 0 38 30·04 | 1 19 58·2 | 2 16·08 | 17 12·7 |
| 4 11 47·8 | 45 | 0 38 3·15 | 1 16 43·2 | 2 15·45 | 17 6·5 |
| 4 13 11·6 | 100 | 0 38 2·36 | 1 16 37·9 | 2 15·48 | 17 7·1 |
| 6 12 52·3 | 40 | 0 37 33·10 | 1 13 7·6 | 2 14·80 | 17 0·7 |
| 7 13 33·6 | 31 | 0 37 17·62 | 1 11 17·1 | 2 14·44 | 16 57·3 |

*Errors of Tabular Place of Saturn.*

The positions of the satellite given above depend on the positions of certain stars taken from catalogues of the Astronomische Gesellschaft, and will be affected by the mean error in the places of these stars. To eliminate this it is necessary to determine the position of Saturn referred to the same stars.

For this purpose a number of photographs of Saturn were taken with the 26-inch refractor, using the occulting shutter. By this means good measurable images of Saturn were obtained, together with sufficiently exposed images of the reference stars. The field of the 26-inch on a 16 c.m. plate is one square degree, and the reference stars in this area were measured together with Saturn. The adopted places of these stars were deduced from measures made on the Astrographic reference plate, the constants of the reference plate being, as mentioned above, derived from all the available stars in the Astronomische Gesellschaft catalogues. The places of these reference stars, and therefore the deduced place of Saturn, will be affected by the mean systematic error of all the catalogue stars on the reference plate, and will thus be comparable with the deduced places of the satellite.

Five photographs were selected for measurement. From two to four images of Saturn and of each of the stars were measured on each plate in the Astrographic micrometer, and the following are the results obtained :—

*Errors of N.A. Tabular Place of Saturn.*

| | Tab.—Obs. R.A. | Tab.—Obs. Dec. |
|---|---|---|
| | s | ″ |
| Aug. 10 | −·10 | +0·0 |
| 25 | −·08 | +0·2 |
| 25 | −·08 | +0·1 |
| 27 | −·07 | −0·2 |
| 30 | −·07 | −0·1 |
| Mean | −·08 | +0·0 |

Corrections for these mean errors have been applied to the tabular places of Saturn as already stated.

It may be mentioned that on six nights, August 25, 28, 29, 30,

September 2 and 4, duplicate photographs with Saturn at the centre of the field were taken with exposures of 90 to 100 minutes in a search for Themis, which could not be found on these photographs, although taken under favourable conditions.

*Royal Observatory, Greenwich:*
1909 *January* 8.

---

*Observations of Occultations of Stars by the Moon, made at the Royal Observatory, Greenwich, in the Year* 1908.

(*Communicated by the Astronomer Royal.*)

| Day, 1908. | | Phenomenon. | Telescope. | Power. | Moon's Limb. | Mean Solar Time of Observation. | Observer. |
|---|---|---|---|---|---|---|---|
| | | | | | | h m s | |
| Mar. 12 | (*a*) | Disapp. B.D. +22°, 1780 | Astrographic Equatorial | 225 | Dark | 6 50 53·76 | A. L. |
| 13 | | ,, Piazzi VIII., 121 | ,, ,, | 225 | ,, | 6 55 31·78 | W. S. |
| 13 | (*b*) | ,, B.D. +20°, 2154 | ,, ,, | 225 | ,, | 7 5 27·16 | W. S. |
| 13 | | ,, 39 Cancri | ,, ,, | 225 | ,, | 7 13 54·57 | W. S. |
| 13 | | ,, 40 Cancri | ,, ,, | 225 | ,, | 7 20 0·08 | W. S. |
| 13 | | ,, ,, | Sheepshanks Equatorial | 100 | ,, | 7 20 0·03 | J. E. |
| 13 | | ,, B.D. +20°, 2161 | Astrographic Equatorial | 225 | ,, | 7 20 7·76 | W. S. |
| 13 | (*b*) | ,, B.D. +20°, 2169 | ,, ,, | 225 | ,, | 7 25 50·62 | W. S. |
| 13 | | ,, B.D. +20°, 2174 | ,, ,, | 225 | ,, | 7 50 25·49 | W. S. |
| 13 | | ,, B.D. +20°, 2179 | ,, ,, | 225 | ,, | 8 1 41·94 | W. S. |
| 13 | | ,, Bradley, 1228 | Sheepshanks Equatorial | 100 | ,, | 8 28 7·63 | W. S. |
| 23 | | Reapp. Bradley, 2162 | Astrographic Equatorial | 225 | ,, | 16 48 9·05 | J. S. |
| Apr. 7 | | Disapp. B.D. +22°, 1420 | ,, ,, | 225 | ,, | 7 8 7·91 | W. S. |
| 7 | | ,, B.D. +22°, 1427 | ,, ,, | 225 | ,, | 7 45 14·49 | W. S. |
| 7 | (*c*) | ,, B.D. +22°, 1432 | ,, ,, | 225 | ,, | 8 33 20·11 | W. S. |
| 7 | | ,, B.D. +22°, 1439 | ,, ,, | 225 | ,, | 8 34 45·87 | W. S. |
| 7 | | ,, B.D. +22°, 1446 | ,, ,, | 225 | ,, | 9 1 55·22 | W. S. |
| 7 | | ,, B.D. +22°, 1448 | ,, ,, | 225 | ,, | 9 23 10·92 | W. S. |
| 7 | | ,, B.D. +22°, 1450 | ,, ,, | 225 | ,, | 9 37 5·95 | W. S. |
| 7 | | ,, B.D. +22°, 1453 | ,, ,, | 225 | ,, | 9 58 6·70 | W. S. |
| 7 | | ,, B.D. +22°, 1451 | ,, ,, | 225 | ,, | 9 59 55·91 | W. S. |
| 8 | | ,, B.D. +22°, 1721 | ,, ,, | 225 | ,, | 8 45 54·68 | W. S. |
| May 9 | | ,, B D. +13°, 2280 | ,, ,, | 225 | ,, | 10 8 4·67 | W. S. |
| Sept. 17 | (*d*) | Reapp. B.D. +22°, 1213 | Altazimuth | 160 | ,, | 11 49 39·71 | A. C. |
| Nov. 9 | (*e*) | ,, ε Tauri | Sheepshanks Equatorial | 100 | ,, | 9 20 21·68 | R. C. |

(*a*) Observation doubtful, clouds passing. (*b*) Faint.
(*c*) Star glided along limb.
(*d*) Star reappeared while Moon was being observed with instrument in position 70° r.
(*e*) Observation doubtful, could not hear clock.

The apertures of the telescopes used are as follows:—

| | |
|---|---|
| Astrographic Equatorial (guiding telescope) | 10 inches. |
| Altazimuth | 8 ,, |
| Sheepshanks Equatorial | $6\frac{3}{4}$ ,, |

The initials A. C., J. S., W. S., J. E., R. C., A. L., are those of Mr. Crommelin, Mr. Storey, Mr. Stevens, Mr. J. Evans, Mr. Cullen, and Mr. Lyne.

*Observations of Comet c* 1908 (*Morehouse*). By R. C. Johnson. (Plate 12.)

This paper consists of notes of the dates upon which negatives of comet *c* 1908 were obtained at West Kirby with a mirror of $6\frac{3}{8}$ inches in diameter and 27·9 inches focal length, denoted by R, and a Voigtländer portrait combination of $4\frac{1}{8}$ inches in diameter and 13·6 inches focal length, denoted by V.

Photographs taken with such small apertures are not intended to compete with the work done by large instruments, but the dates are submitted, with the hope that possibly some of the negatives secured in November, when the comet's altitude was very small, may be found useful in the investigation of the movement of cometary matter.

| 1908. | G. M. T. of Mean Exposure. | Time of Exposure in minutes. | Age of ☽ at midnight. | Instrument | State of Sky. | Notes. |
|---|---|---|---|---|---|---|
| | h m | | days | | | |
| 1 Sept. 17 | 9 15 | 30 | | V | hazy | Tail 2°. |
| 2 21 | | 7 | | V | hazy | Tail v. narrow, 7° in length. |
| 3 22 | | 50 | | V R | | |
| 4 29 | 8 42 | 45 | | V R | very clear | Very fine aurora; made observatory quite light. |
| 5 30 | 8 28 | 20 | 5·38 | V R | | |
| 6 30 | 9 30 | 30 | | V R | | Tail $8\frac{1}{2}$°. |
| 7 Oct. 3 | 10 42 | 45 | 8·38 | V R | | Tail $2\frac{3}{4}$°; several branches widening out to 25°. |
| 8 13 | 7 35 | 20 | 18·38 | V R | slight haze | Tail 5° long. |
| 9 17 | 10 15 | 20 | | R | | |
| 9a 17 | 10 20 | 10 | | V | | ☌ seen distinctly on X wire of following telescope for first time; tail $7\frac{1}{2}$°+. |
| 10 21 | 8 53 | 32 | | V R | very clear | |
| 11 24 | 9 52 | 40 | | V R | very clear | Tail $10\frac{1}{4}$°; slightly curved, with many fine streamers on N. side of axis. |

Comet 1908 c (Morehouse), Nov. 15 d. 6 h. 3 m G.M.T.
6¾-in. Reflector. R. C. Johnson.

| 1908. | G.M.T. of Mean Exposure. | Time of Exposure in minutes. | Age of ☽ at midnight. | Instrument. | State of Sky. | Notes. |
|---|---|---|---|---|---|---|
| | h m | | days | | | |
| 12 Oct. 25 | | 30 | | V R | very clear | |
| 13 Nov. 3 | 7 0 | 16 | 9·72 | V R | haze | V plate blackened with ☽. 5° tail. |
| 14 4 | 6 32 | 16 | 10·72 | V R | | E. wind, but clearer than on Nov. 3. |
| 15 8 | 6 13 | 21 | 14·72 | R | | Knot in tail $3\frac{1}{2}°$ from nucleus; several fine rays at nucleus. |
| 15a 8 | 6 15 | 17 | 14·72 | V | | |
| 16 8 | 7 36 | 33 | 14·72 | R | | |
| 16a 8 | 7 33 | 28 | 14·72 | V | S.E. wind | Slight haze. |
| 17 9 | 6 32 | 30 | 15·72 | V R | | Tail straight, 6°. |
| 18 15 | 6 3 | 42 | 21·72 | V R | very clear | $9\frac{1}{2}°$ tail; much detail. Not quite dark at beginning of exposure. |
| 19 18 | 6 26 | 10 | | V R | clear intervals | Delayed by cloud. $10\frac{1}{2}°$ tail. |
| 20 25 | 6 18 | 34 | | V R | clear | Slight horizon haze at end. Tail 8°+. |
| 21 29 | 5 51 | 22 | | R | clear | but not dark. |

In the 5th column of table the Moon's age at midnight is stated when definition was interfered with.

## *Micrometrical Measures of Double Stars (Sixth Series).*

By the Rev. T. E. Espin, M.A.

The stars measured in 1908 are mainly those of *h* which hitherto have remained unmeasured. The working list contains 203 stars of *h* between +40° and +50° in declination, and 107 of these have now been measured. In the list of various stars will be found ten pairs entered in column 1 as Arg. VI., followed by a number. These are from a list of fifty-three pairs collected from vol. vi. of the Bonn Observations which had not been noted previously to Argelander's Observations. Several of these have been rediscovered by modern observers. In some cases *h*'s place is in error, and the pair has then been searched for, and when identified the correct place is given, and an asterisk is affixed to the star. The star *h* 2063 is identical with A 944 A C, *h*'s place being 1m. in R.A. too great; the star *h* 2510 R.A. 9h 46m 35s Decl. +49° 22′ (1880) was searched for on the nights of March 23 and April 7, without result.

*h*'s *Stars.*

| *h*. | R.A. 1880. | Decl. | P. | D. | Mags. | Nights. | Date. 1908. |
|---|---|---|---|---|---|---|---|
| | h m | ° ′ | ° | ″ | | | |
| 1036 | 0 28·1 | +42 13 | 81·9 | 6·22 | 9·2, 9·8 | 2 | ·054 |
| 1987 | 28·8 | 42 24 | 348·5 | 21·62 | 8·5, 11·0 | 3 | ·334 |
| 1044 | 33·7 | 43 3 | 318·4 | 21·84 | 9·0, 9·2 | 3 | ·338 |
| 1062* | 53·0 | 48 56 | 106·4 | 9·90 | 8·8, 10·7 | 3 | ·042 |
| 1071 | 1 1·3 | 46 46 | 118·4 | 14·72 | 9·0, 11·5 | 2 | ·024 |
| 2057 | 28·9 | 45 45 | 42·9 | 11·02 | 9·0, 11·0 | 2 | ·015 |
| 2089 | 44·1 | 42 53 | 305·2 | 29·26 | 8·6, 9·0 | 3 | ·047 |
| 2117 | 2 4·7 | 44 6 | 30·1 | 8·00 | Var. | 3 | ·030 AB |
| | | | 199·8 | 13·50 | C=10·4 | 3 | ·030 AC |
| | | | 285·1 | 24·43 | D=11·7 | 3 | ·030 AD |
| 2137 | 20·7 | 42 42 | 132·8 | 26·22 | 8·6, 9·9 | 3 | ·023 |
| 2141 | 22·2 | 44 57 | 141·7 | 10·95 | 11·0, 11·6 | 2 | ·068 |
| 2139 | 22·6 | 52 38 | 298·0 | 3·52 | 8·5, 8·6 | 2 | ·918 AB |
| | | | 6·5 | 20·43 | C=12·0 | 2 | ·918 AC |
| 2147 | 28·9 | 45 32 | 166·5 | 9·82 | 9·3, 10·5 | 2 | ·055 |
| 2160 | 43·5 | 47 33 | 268·3 | 8·02 | 10·1, 11·2 | 2 | ·063 |
| 2162 | 46·9 | 43 3 | 38·5 | 12·72 | 9·4, 9·6 | 3 | ·040 AB |
| 2163 | | | 6·6 | 10·30 | 11·3, 12·7 | 3 | ·040 CD |
| | | | 52·8 | 68·78 | | 3 | ·040 AC |
| 2167 | 53·1 | 44 25 | 33·9 | 36·87 | 8·8, 9·5 | 3 | ·015 |
| 2297* | 6 5·3 | 48 38 | 17·8 | 8·60 | 10·0, 11·5 | 1 | ·066 |

*h*'s *Stars*—continued.

| *h*. | R.A. 1880. h m | Decl. ° ′ | P. ° | D. ″ | Mags. | Nights. | Date. 1908. |
|---|---|---|---|---|---|---|---|
| 2342 | 6 43·9 | 49 40 | 49·3 | 21·60 | 8·3, 13·8 | 2 | ·083 AB |
| | | | 74·6 | 27·90 | 14·2 | 2 | ·083 AC |
| 1160 | 8 15·2 | 47 9 | 150·4 | 28·75 | 8·8, 12·0 | 2 | ·250 |
| 1161 | 25·2 | 46 20 | 28·2 | 19·85 | 9·5, 13·5 | 1 | ·255 |
| 2457 | 30·5 | 47 54 | 263·7 | 14·25 | 10·0, 13·0 | 1 | ·255 |
| | | | 289·9 | 28·40 | 13·3 | 1 | ·255 |
| 1165 | 9 8·4 | 45 26 | 116·0 | 24·67 | 8·5, 12·2 | 2 | ·235 |
| 2488 | 8·7 | 48 1 | 226·2 | 9·40 | 12·5, 12·8 | 1 | ·255 |
| 2503 | 36·6 | 49 2 | 155·7 | 36·52 | 9·2, 9·3 | 2 | ·238 |
| 1171 | 40 7 | 47 20 | 183·6 | 15·57 | 9·5, 11·2 | 2 | ·238 |
| 5478 | 53·9 | 45 34 | 41·4 | 11·00 | 9·5, 11·0 | 1 | ·266 |
| 2522 | 10 4·5 | 48 27 | 147·3 | 28·82 | 9·0, 11·2 | 2 | ·253 |
| 2546 | 45·0 | 48 42 | 56·4 | 6·97 | 9·5, 10·8 | 3 | ·280 |
| 2554 | 56·9 | 44 58 | 278·3 | 49·02 | 7·7, 8·9 | 2 | ·253 |
| 1184 | 59·1 | 46 38 | 30·7 | 15·90 | 9·5, 14·5 | 2 | ·253 |
| | | | 105·9 | 30·62 | 12·7 | 2 | ·253 |
| 2622 | 12 48·8 | 43 28 | 329·3 | 26·25 | 9·2, 12·1 | 2 | ·260 |
| 2623 | 49·0 | 43 33 | 156·3 | 23·35 | 9·3, 12·3 | 2 | ·260 |
| 1223 | 53·7 | 43 24 | 191·5 | 20·57 | 8·6, 10·7 | 2 | ·360 |
| 2632 | 57·6 | 47 22 | 347·1 | 30·75 | 9·1, 13·2 | 2 | ·388 |
| 2634 | 57·9 | 48 23 | 60·1 | 30·85 | 9·1, 11·2 | 3 | ·390 |
| 2642 | 13 2·7 | 49 55 | 173·8 | 9·65 | 9·3, 9·8 | 1 | ·402 |
| 2667 | 32·8 | 48 52 | 3·2 | 13·00 | 9·5, 9·7 | 2 | ·349 |
| 2675 | 38·1 | 47 46 | 295·2 | 10·90 | 11·3, 12·3 | 3 | ·390 |
| 2680 | 41·3 | 46 0 | 153·9 | 22·27 | 9·3, 10·5 | 2 | ·444 |
| 2697 | 56·2 | 46 59 | 293·4 | 23·75 | 9·2, 11·0 | 2 | ·389 |
| 2720 | 14 19·5 | 47 1 | 31·3 | 21·12 | 9·0, 13·3 | 3 | ·384 |
| 2752 | 47·1 | 45 6 | 121·9 | 5·58 | 8·3, 9·0 | 3 | ·372 |
| 2759 | 53·2 | 45 59 | 92·1 | 12·90 | 9·5, 13·7 | 2 | ·383 |
| 2768 | 15 4·4 | 45 37 | 114·8 | 20·75 | 10·2, 11·1 | 2 | ·383 |
| 2781 | 22·6 | 49 38 | 325·5 | 22·30 | 9·3, 11·5 | 2 | ·396 |
| 1274 | 26·4 | 42 39 | 311·0 | 6·07 | 9·3, 9·5 | 2 | ·446 |
| 1418 | 19 30·0 | 49 46 | 10·4 | 29·95 | 8·9, 9·1 | 2 | ·861 |
| 1419 | 30·2 | 47 51 | 38·6 | 6·95 | 10·5, 12·0 | 2 | ·861 |
| 1478(1) | 59·8 | 43 40 | 221·8 | 15·97 | 9·3, 9·4 | 2 | ·831 |
| (2) | 20 0·3 | 43 39 | 337·5 | 8·00 | 11·0, 11·5 | 1 | ·832 |
| 1488* | 6·9 | 45 25 | 121·5 | 9·97 | 10·1, 10·2 | 2 | ·840 |
| 1558 | 34·8 | 48 5 | 205·9 | 4·45 | 9·0, 11·0 | 2 | ·901 |
| 1666 | 21 31·1 | 43 0 | 234·9 | 9·32 | 10·1, 10·8 | 2 | ·858 |

*Notes.*

*h* 1044 *h* 324°·5, Cin : 1879·6, 318°·8, 21″·9.

*h* 1062 This is B.D. + 48°·311. *h*'s Decl. requires a correction of + 20′.

*h* 2117 Observations on fifteen nights show that either A or B is variable to the extent of about half a magnitude, and that the period is short.

*h* 2297 *h*'s place requires a correction of + 40 sec. in R.A.

*h* 2342 Both *comites* are difficult to measure on account of their faintness.

*h* 2488 The N. star is distinctly the brighter.

*h* 1184 The nearer *comes* is extremely difficult to measure.

*h* 2667 This is B.D. + 48°, 3136.

*h* 2752 The only measures are—

| | P. | D. | Date. |
|---|---|---|---|
| | ° | ″ | |
| *h* | 131·2 | | |
| Ma | 126·3 | 5·44 | 1843·32 |
| *β* | 126·5 | 5·84 | 1878·26 |
| Es | 121·9 | 5·58 | 1908·37 |

*h* 1274 This is B.D. + 42°, 2604, and *h*'s place requires a correc- of + 20′ in declination.

*h* 1478 The second of these two pairs agrees with *h*'s place, but his angle of 228° ±, which he says is estimated from a doubtful diagram, agrees with the first. Probably some confusion arose between the two pairs in the same low-powered field. The actual places of the pairs and *h*'s double are as follows :—

| | R.A. 1855·0. | | | Decl. | |
|---|---|---|---|---|---|
| | h | m | s | ° | ′ |
| (1) | 19 | 58 | 58·3 | + 43 | 35·1 |
| (2) | | 59 | 29·5 | 43 | 34·2 |
| *h* 1478 | | 59 | 28·1 | 43 | 35·5 |

*h* 1488 This is B.D. + 45°·3079.

*Various Stars.*

| Name. | R.A. 1880. | | Decl. | | P. | D. | Mags. | Nights. | Date. |
|---|---|---|---|---|---|---|---|---|---|
| | h | m | ° | ′ | ° | ″ | | | 1908. |
| *η* Cassiopeiæ ... | 0 | 41·7 | + 57 | 11 | 239·8 | 5·80 | | 3 | ·723 |
| Σ 70 ... ... | | 46·9 | 52 | 2 | 245·5 | 7·90 | | 2 | ·884 AB |
| | | | | | 86·8 | 1·90 | 11·0, 11·5 | 3 | ·889 CD |
| | | | | | 148·2 | 80·55 | | | ·884 AC |
| Espin 45 ... | | 54·5 | 48 | 54 | 245·4 | 7·68 | 6·8, 10·7 | 3 | ·055 |

*Various Stars*—continued.

| Name. | R.A. 1880. h m | Decl. ° ′ | P. ° | D. ″ | Mags. | Nights. | Date. 1908. | |
|---|---|---|---|---|---|---|---|---|
| Arg. VI. 3 | ... 1 8·8 | 42 17 | 293·5 | 12·48 | 9·2, 9·5 | 3 | ·871 | BC |
| | | | 101·8 | 35·10 | A= 8·3 | 3 | ·871 | AB |
| | | | 97·2 | 22·87 | | 3 | ·871 | AC |
| Arg. VI. 4 | ... 19·6 | 48 31 | 158·1 | 11·13 | 9·5, 10·2 | 2 | ·824 | |
| β— ... | ... 43·9 | 52 14 | 265·5 | 3·92 | 9·5, 10·1 | 3 | ·777 | BC |
| | | | 342·5 | 50·16 | A= 9·0 | 3 | ·777 | AB |
| A 1527 | ... 54·0 | 43 5 | 238·1 | 4·33 | 8·2, 11·1 | 3 | ·061 | |
| Arg. VI. 7 | ... 2 17·1 | 51 50 | 244·1 | 17·03 | 8·8, 9·2 | 2 | ·854 | |
| A.G. (Bonn) | ... 26·1 | 43 38 | 290·7 | 15·45 | 9·1, 9·2 | 2 | ·041 | AB |
| | | | 35·4 | 18·70 | C= 10·8 | 2 | ·072 | AC |
| | | | 248·5 | 32·82 | D= 14·0 | 2 | ·072 | AD |
| A.G. (Harvard) | 3 53·3 | 54 7 | 325·4 | 19·38 | 8·8, 9·2 | 2 | ·984 | |
| Σ 599 ... | ... 4 42·4 | 44 46 | 335·3 | 9·83 | 8·0, 9·0 | 3 | ·079 | AB |
| | | | 67·2 | 14·55 | C= 14·0 | 3 | ·079 | BC |
| A 1571 | ... 5 51·1 | 43 58 | 91·4 | 4·52 | 8·0, 12·2 | 3 | ·053 | |
| Hu 711 | ... 7 46·4 | 48 28 | 199·9 | 4·06 | 7·9, 11·2 | 4 | ·158 | |
| Σ 1242 ... | ... 8 27·6 | 47 32 | 170·7 | 2·64 | 8·8, 9·0 | 1 | ·266 | |
| Σ 1289 ... | ... 46·7 | 44 3 | 3·4 | 3·74 | 7·5, 8·0 | 1 | ·266 | |
| Σ 1358 ... | ... 9 23·1 | 45 12 | 161·9 | 23·45 | 7·5, 8·5 | 1 | ·266 | |
| Σ 1394 ... | ... 49·6 | 46 29 | 244·3 | 3·79 | 8·5, 8·8 | 1 | ·266 | |
| Ku 39 ... | ... 11 30·8 | 48 8 | 24·6 | 2·54 | 9·5, 9·6 | 1 | ·266 | |
| A.G. 190 | ... 13 30·8 | 50 16 | 10·3 | 2·87 | 9·0, 9·2 | 2 | ·444 | |
| Swift ... | ... 57·7 | 46 55 | 7·5 | 2·52 | 9·0, 9·2 | 5 | ·369 | |
| Ku 54 ... | ... 16 39·3 | 44 8 | 96·9 | 9·07 | 8·4, 9·0 | 3 | ·389 | |
| O. Stone | ... 17 25·2 | 46 22 | 184·8 | 7·60 | 9·3, 9·4 | 1 | ·381 | |
| A 1880 | ... 35·6 | 52 40 | 114·7 | 2·57 | 8·7, 10·2 | 3 | ·481 | |
| A 1886 | ... 57·4 | 53 16 | 340·3 | 4·75 | 8·7, 9·2 | 2 | ·563 | |
| A.G. (Harvard) | 18 9·1 | 51 8 | 90·0 | 14·09 | 9·2, 10·0 | 1 | ·832 | BC |
| | | | 340·2 | 14·70 | A= 8·5 | 1 | ·832 | AC |
| | | | 306·3 | 23·72 | | 1 | ·832 | AB |
| Arg. VI. 35 | ... 20·9 | 51 36 | 202·4 | 2·40 | 8·3, 8·4 | 4 | ·504 | AB |
| (Espin 187) | ... | | 118·6 | 85·44 | C= 9·0 | 3 | ·484 | AC |
| Arg. VI. 37 | ... 19 5·5 | 44 51 | 236·6 | 17·56 | 9·3, 11·3 | 3 | ·777 | AB |
| | | | 334·0 | 26·38 | C= 9·7 | 3 | ·777 | BC |
| Arg. VI. 40 | ... 42·1 | 46 5 | 163·7 | 26·67 | 8·2, 12·0 | 1 | ·753 | AB |
| | | | 317·8 | 29·84 | C= 9·3 | 2 | ·720 | AC |
| Arg. VI. 41 | ... 50·1 | 49 20 | 270·0 | 15·25 | 9·3, 9·5 | 2 | ·750 | |
| U Cygni | ... 20 15·9 | 47 31 | 51·4 | 62·52 | Var. 8·0 | 2 | ·749 | |
| Arg. VI. 46 | ... 21·4 | 41 31 | 312·9 | 46·74 | 9·0, 9·5 | 1 | ·769 | |

*Various Stars*—continued.

| Name. | R.A. 1880. h m | Decl. ° ′ | P. ° | D. ″ | Mags. | Nights. | Date. 1908. |
|---|---|---|---|---|---|---|---|
| Espin 139 ... | 21 17·3 | 52 52 | 349·0 | 6·00 | 9·4, 10·6 | 2 | ·924 |
| A.G. (Bonn) ... | 30·0 | 42 50 | 112·7 | 14·13 | 8·5, 9·0 | 2 | ·858 |
| Arg. VI. 49 ... | 44·3 | 42 8 | 341·4 | 25·67 | 9·2, 9·6 | 3 | ·778 |
| Arg. VI. 52 ... | 22 31·2 | 45 25 | 135·9 | 12·50 | 9·2, 10·2 | 2 | ·762 AB |
| | | | 215·8 | 16·66 | C= 9·8 | 3 | ·764 AC |
| A.G. (Harvard) | 43·5 | 53 31 | 135·7 | 2·95 | 8·5, 8·8 | 2 | ·955 |
| β— ... | 45·5 | 52 28 | 172·3 | 4·40 | 9·2, 10·0 | 2 | ·932 AB |
| | | | 224·9 | 25·47 | C= 9·5 | 2 | ·932 AC |

*Notes.*

Σ 70. There are no other measures of the distant double *comes* which had been previously seen by β.

β—. This was measured as new, but had been previously measured by β, under *h* 2085 Gen. Cat., No. 994.

Σ 599. The *comes* C had been already measured by β.

A 1527, A 1571. These stars were detected and measured before Professor Aitken's results reached me.

O. Stone. This is B.D. +46°, 2316.

A. 1880, A 1886. The discovery of these two pairs and measures of them were published in the Journal of the R.A.S. of Canada, vol. ii. No. 5 (September–October). This appeared when Professor Aitken's thirteenth list of New Double Stars was already in type. To save duplication, I have struck them out of my list of new pairs.

U Cygni. All the measures are—

| | Date. | P. ° | D. ″ | Nights. |
|---|---|---|---|---|
| Tarrant | 1886·901 | 51·2 | 62·68 | 4 |
| Espin | 1892·797 | 52·3 | 62·70 | 2 |
| ,, | 1908·749 | 51·4 | 62·52 | 2 |

β—. Measured as new in the first instance, but it had already been measured by β (see Gen. Cat., vol. ii. p. 1004).

## *New Double Stars.* By the Rev. T. E. Espin, M.A.

| No. | B.D. | R.A. 1900. | Decl. | P. | D. | Mags. | | Nights. | Date. |
|---|---|---|---|---|---|---|---|---|---|
| | ° | h m | ° ′ | ° | ″ | | | | 1908. |
| 610 | +54, 3111 | 0 1·2 | +54 27 | 203·4 | 4·20 | 9·4 | 11·2 | 2 | ·941 |
| 611 | +53, 3289 | 1·6 | 53 56 | 292·2 | 10·50 | 8·4 | 10·1 | 2 | ·872 |
| 612 | +54, 32 | 13·3 | 54 19 | 266·0 | 2·90 | 9·3 | 9·5 | 2 | ·988 |
| 613 | +52, 79 | 24·0 | 52 50 | 270·6 | 8·25 | 8·3 | 11·8 | 3 | ·744 |
| 614 | +52, 182 | 46·6 | 53 11 | 86·8 | 2·23 | 9·5 | 9·8 | 3 | ·748 |
| 615 | +53, 190 | 53·1 | 53 22 | 267·4 | 3·27 | 9·1 | 11·7 | 2 | ·802 |
| 616 | +53, 223 | 1 0·6 | 53 53 | 278·1 | 3·35 | 9·4 | 10·3 | 3 | ·846 |
| | +53, 326 | 26·3 | 54 14 | 195·4 | 16·15 | 8·6 | 8·8 | 2 | ·907 |
| | +53, 327 | 26·8 | 53 46 | 28·1 | 15·33 | 8·7 | 9·0 | 2 | ·907 |
| 617 | +54, 324 | 29·7 | 54 27 | 304·9 | 3·27 | 9·1 | 9·2 | 2 | ·852 |
| 618 | +53, 491 | 2 10·5 | 53 41 | 173·1 | 3·65 | 9·3 | 11·6 | 3 | ·895 |
| 619 | +45, 600 | 15·1 | 45 57 | 243·6 | 5·55 | 8·0 | 10·7 | 2 | ·828 |
| 620 | +53, 546 | 28·5 | 53 15 | 212·7 | 6·12 | 8·5 | 12·0 | 2 | ·916 |
| 621 | +53, 601 | 52·5 | 53 56 | 354·1 | 8·25 | 8·4 | 11·5 | 3 | ·917 |
| 622 | +53, 680 | 3 28·1 | 53 34 | 34·1 | 4·62 | 9·1 | 9·9 | 3 | ·915 |
| 623 | +53, 705 | 43·7 | 53 15 | 331·2 | 8·02 | 8·5 | 13·9 | 2 | ·941 |
| 624 | *h* 2770 | 15 9·8 | 47 14 | 49·6 | 2·30 | 10·0 | 10·1 | 3 | ·386 AB |
| | | | | 129·3 | 16·57 | C = | 10·5 | 2 | ·383 AC |
| | +49, 2363 | 9·5 | 48 57 | 343·0 | 25·75 | 7·0 | 10·2 | 2 | ·401 |
| 625 | +44, 2454 | 17·7 | 44 40 | 252·9 | 1·57 | 9·3 | 10·3 | 2 | ·413 |
| 626 | | 38·0 | 50 28 | 275·0 | 7·52 | 9·2 | 9·2 | 2 | ·469 |
| 627 | +51, 2073 | 16 16·0 | 51 34 | 286·5 | 11·07 | 8·6 | 9·8 | 2 | ·472 |
| 628 | +52, 1959 | 17·3 | 52 0 | 265·7 | 3·37 | 8·8 | 12·0 | 2 | ·472 |
| 629 | +52, 1961 | 18·0 | 52 4 | 93·2 | 10·20 | 8·1 | 13·0 | 2 | ·472 |
| 630 | +54, 1813 | 25·4 | 54 28 | 44·7 | 5·47 | 9·0 | 13·5 | 2 | ·517 |
| 631 | +43, 2621 | 32·1 | 43 38 | 162·1 | 9·77 | 8·2 | 11·0 | 2 | ·401 |
| 632 | | 38·2 | 50 25 | 102·9 | 1·75 | 9·3 | 10·0 | 2 | ·469 |
| 633 | +42, 2783 | 58·4 | 42 52 | 258·9 | 6·30 | 7·0 | 11·5 | 2 | ·412 |
| 634 | +42, 2789 | 17 0·8 | 42 20 | 98·0 | 1·80 | 9·4 | 10·0 | 2 | ·444 |
| 635 | +52, 2075 | 33·0 | 52 14 | 241·7 | 5·77 | 8·9 | 12·0 | 2 | ·568 |
| 636 | | 33·4 | 41 46 | 124·4 | 2·38 | 9·2 | 9·7 | 3 | ·517 |
| 637 | +54, 1898 | 34·2 | 54 29 | 296·4 | 3·70 | 9·2 | 9·3 | 2 | ·489 |
| 638 | +54, 1902 | 38·2 | 54 14 | 188·2 | 2·57 | 9·2 | 11·0 | 4 | ·530 |
| 639 | +56, 2019 | 44·3 | 56 16 | 76·1 | 8·00 | 8·8 | 9·9 | 2 | ·598 |
| 640 | +54, 1931 | 58·8 | 54 53 | 75·6 | 7·90 | 8·6 | 9·2 | 2 | ·552 |
| 641 | +54, 1937 | 18 2·8 | 54 34 | 66·7 | 1·95 | 9·2 | 9·4 | 3 | ·602 |
| 642 | +51, 2327 | 8·5 | 51 38 | 273·8 | 6·75 | 9·0 | 13·0 | 2 | ·650 |
| 643 | +55, 2039 | 10·9 | 55 54 | 50·3 | 3·40 | 9·0 | 11·2 | 2 | ·565 |
| 644 | +52, 2167 | 11·2 | 52 27 | 32·0 | 9·97 | 8·5 | 14·0 | 2 | ·640 |
| 645 | +53, 2054 | 11·7 | 53 39 | 89·1 | 2·75 | 8·2 | 12·0 | 3 | ·560 |
| 646 | +52, 2175 | 12·9 | 52 5 | 197·2 | 9·88 | 8·0 | 13·8 | 3 | ·654 |
| 647 | +50, 2561 | 15·8 | 50 47 | 293·4 | 2·80 | 9·0 | 11·3 | 3 | ·776 |
| 648 | +52, 2197 | 19·8 | 52 18 | 5·2 | 4·68 | 9·0 | 13·7 | 4 | ·666 |
| 649 | +53, 2074 | 19·9 | 53 31 | 316·5 | 8·32 | 9·2 | 11·5 | 2 | ·624 |

| No. | B.D. | R.A. 1900. | Decl. | P. | D. | Mags. | | Nights. | Date. 1908. |
|---|---|---|---|---|---|---|---|---|---|
| | ° | h m | ° ′ | ° | ″ | | | | |
| 650 | +52, 2305 | 18 53·5 | 52 33 | 329·5 | 3·22 | 8·8 | 12·0 | 3 | ·752 AB |
| | | | | 193·6 | 26·79 | C= | 10·2 | 2 | ·721 AC |
| 651 | +51, 2476 | 57·8 | 51 58 | 140·1 | 5·87 | 9·2 | 9·5 | 2 | ·668 |
| 652 | +50, 2758 | 19 14·0 | 50 41 | 80·0 | 7·05 | 9·2 | 11·2 | 2 | ·610 |
| 653 | +53, 2241 | 23·8 | 53 57 | 284·6 | 11·20 | 9·0 | 9·0 | 2 | ·551 |
| 654 | +54, 2152 | 26·0 | 54 34 | 191·5 | 7·60 | 7·2 | 12·5 | 3 | ·641 |
| 655 | +54, 2182 | 32·6 | 54 57 | 57·7 | 2·95 | 10·2 | 11·7 | 5 | ·681 BC |
| | | | | 128·9 | 67·07 | A= | 9·1 | 3 | ·636 AB |
| 656 | *h* 1427 | 34·4 | 46 4 | 286·5 | 8·27 | 10·0 | 10·4 | 2 | ·841 AB |
| | | | | 246·2 | 2·75 | 11·2 | 11·5 | 2 | ·841 CD |
| | | | | 293·1 | 39·13 | | | 2 | ·841 AC |
| 657 | +52, 2489 | 39·7 | 53 5 | 113·9 | 2·70 | 9·4 | 11·0 | 2 | ·562 |
| 658 | +52, 2521 | 44·0 | 52 53 | 316·3 | 5·18 | 8·6 | 14·0 | 3 | ·571 AB |
| | | | | 296·7 | 8·10 | 12·0 | 13·0 | 2 | ·564 CD |
| | | | | 90·4 | 31·41 | | | 3 | ·571 AC |
| 659 | +54, 2230 | 47·3 | 54 39 | 160·7 | 5·83 | 9·2 | 9·3 | 3 | ·744 |
| 660 | +51, 2821 | 20 14·5 | 51 42 | 288·2 | 9·05 | 9·0 | 9·0 | 5 | ·602 BC |
| | | | | 304·1 | 30·41 | A= | 8·9 | 4 | ·568 AB |
| | +51, 2833 | 16·1 | 51 53 | 270·1 | 24·67 | 8·3 | 8·4 | 1 | ·572 |
| 661 | +44, 3474 | 24·8 | 44 38 | 163·5 | 8·60 | 8·9 | 11·2 | 2 | ·651 |
| 662 | +44, 3485 | 26·1 | 44 56 | 145·6 | 6·60 | 9·0 | 11·2 | 2 | ·774 |
| 663 | +46, 2952 | 27·1 | 46 14 | 111·8 | 5·35 | 9·5 | 11·2 | 2 | ·916 |
| 664 | +46, 2965 | 29·2 | 46 14 | 65·9 | 5·20 | 8·9 | 12·8 | 3 | ·894 |
| 665 | +46, 2973 | 30·4 | 46 57 | 124·6 | 5·45 | 8·9 | 9·2 | 2 | ·918 |
| 666 | | 31·6 | 48 9 | 316·0 | 3·25 | 10·1 | 10·7 | 3 | ·776 |
| 667 | +44, 3525 | 35·2 | 44 17 | 184·3 | 9·92 | 8·7 | 9·2 | 3 | ·640 |
| | +44, 3552 | 39·8 | 44 34 | 297·4 | 40·83 | 8·5 | 8·7 | 3 | ·736 |
| 668 | +44, 3555 | 40·0 | 44 39 | 72·7 | 6·17 | 8·4 | 14·0 | 3 | ·640 AB |
| | | | | 269·9 | 23·30 | C= | 12·7 | 3 | ·640 AC |
| | +44, 3569 | 41·5 | 44 26 | 79·4 | 53·74 | 8·5 | 8·7 | 2 | ·728 |
| 669 | +46, 3054 | 44·8 | 46 31 | 70·2 | 4·58 | 9·2 | 10·4 | 3 | ·798 |
| 670 | +45, 3294 | 46·0 | 45 51 | 81·9 | 6·50 | 9·1 | 9·2 | 2 | ·778 |
| 671 | +46, 3069 | 46·7 | 46 13 | 92·6 | 1·82 | 9·2 | 10·0 | 4 | ·771 |
| 672 | +47, 3195 | 46·9 | 47 11 | 162·5 | 4·05 | 9·2 | 9·3 | 2 | ·918 |
| 673 | | 50·5 | 46 8 | 81·6 | 4·37 | 9·5 | 9·7 | 2 | ·848 |
| 674 | +44, 3636 | 52·3 | 44 25 | 91·5 | 5·05 | 9·4 | 12·0 | 2 | ·668 |
| 675 | +44, 3637 | 52·8 | 44 24 | 107·2 | 3·70 | 9·6 | 11·0 | 2 | ·753 BC |
| | | | | 272·3 | 99·63 | A= | 9·5 | 3 | ·849 AB |
| 676 | +44, 3648 | 54·3 | 45 7 | 284·8 | 6·40 | 9·0 | 11·7 | 4 | ·920 |
| 677 | +44, 3650 | 54·4 | 45 9 | 162·8 | 5·97 | 9·1 | 12·2 | 2 | ·916 |
| 678 | +47, 3379 | 21 17·7 | 47 38 | 116·4 | 4·10 | 9·5 | 11·5 | 2 | ·916 |
| 679 | +47, 3400 | 22·0 | 47 15 | 275·6 | 4·52 | 9·4 | 10·6 | 3 | ·749 |
| 680 | +47, 3415 | 24·2 | 47 23 | 332·5 | 8·80 | 8·6 | 12·0 | 2 | ·798 |
| 681 | +53, 2715 | 46·2 | 53 14 | 46·2 | 2·10 | 9·5 | 9·6 | 3 | ·923 |
| 682 | +47, 3624 | 53·7 | 47 47 | 134·7 | 7·40 | 9·1 | 11·7 | 3 | ·748 |
| 683 | +43, 4098 | 54·6 | 44 13 | 248·7 | 8·85 | 8·4 | 8·9 | 2 | ·705 |
| 684 | +53, 2753 | 55·5 | 54 3 | 273·0 | 4·30 | 9·2 | 12·5 | 2 | ·943 |

| No. | B.D. | R.A. 1900. | Decl. | P. | D. | Mags. | | Nights. | Date. |
|---|---|---|---|---|---|---|---|---|---|
| | ° | h m | ° ′ | ° | ″ | | | | 1908. |
| | +47, 3694 | 22 2·8 | 47 22 | 278·7 | 38·43 | 8·3 | 8·7 | 2 | ·754 |
| 685 | +48, 3621 | 4·4 | 48 43 | 56·7 | 6·04 | 8·5 | 10·3 | 3 | ·912 |
| 686 | +47, 3717 | 6·4 | 47 29 | 159·7 | 7·17 | 10·0 | 10·1 | 3 | ·771 BC |
| | | | | 90·7 | 9·42 | A= | 8·9 | 3 | ·771 AB |
| | | | | 119·3 | 14·24 | | | 2 | ·777 AC |
| | +47, 3720 | 6·6 | 47 25 | 307·6 | 22·10 | 7·0 | 9·3 | 3 | ·767 |
| 687 | +47, 3792 | 21·5 | 47 37 | 269·6 | 4·92 | 9·0 | 9·1 | 2 | ·746 |
| 688 | +46, 3726 | 26·8 | 46 30 | 20·1 | 2·02 | 9·3 | 9·5 | 3 | ·846 |
| 689 | +54, 2800 | 29·3 | 54 15 | 286·6 | 10·33 | 8·6 | 14·0 | 3 | ·814 |
| 690 | +47, 3856 | 32·5 | 47 22 | 258·1 | 11·36 | 8·0 | 11·4 | 3 | ·763 |
| 691 | *h* 1830 | 50·1 | 55 7 | 278·3 | 5·85 | 9·5 | 12·5 | 2 | ·705 AB |
| | | | | 80·3 | 14·27 | C= | 9·6 | 2 | ·705 AC |
| | | | | 276·2 | 20·80 | D= | 13·5 | 1 | ·671 AD |
| 692 | +54, 2894 | 56·2 | 54 16 | 159·4 | 4·00 | 10·0 | 10·5 | 3 | ·914 BC |
| | | | | 252·8 | 94·00 | A= | 8·7 | 3 | ·914 AB |
| | +54, 2899 | 57·8 | 54 22 | 252·6 | 30·83 | 8·8 | 9·0 | 2 | ·916 |
| 693 | +47, 4023 | 58·2 | 47 34 | 37·5 | 5·45 | 9·3 | 11·5 | 2 | ·746 |
| 694 | +47, 4090 | 23 11·5 | 47 28 | 92·3 | 3·52 | 9·5 | 12·0 | 2 | ·750 |
| 695 | +53, 3127 | 14·0 | 53 37 | 312·7 | 5·12 | 8·6 | 9·0 | 3 | ·864 AB |
| | | | | 303·9 | 25·88 | C= | 12·2 | 3 | ·864 AC |
| 696 | +53, 3133 | 14·6 | 53 36 | 230·9 | 2·92 | 8·9 | 11·5 | 3 | ·864 |
| 697 | +54, 2954 | 15·9 | 54 57 | 74·9 | 3·87 | 9·0 | 12·5 | 2 | ·932 BC |
| | | | | 344·1 | 68·26 | A= | 8·5 | 2 | ·941 AB |
| 698 | +47, 4130 | 16·8 | 47 23 | 57·5 | 3·20 | 8·8 | 12·0 | 2 | ·754 |
| 699 | +47, 4238 | 35·8 | 47 17 | 220·4 | 6·88 | 8·7 | 10·2 | 3 | ·843 |
| 700 | +53, 3238 | 45·7 | 53 38 | 34·9 | 14·64 | 6·5 | 10·5 | 2 | ·906 |
| 701 | +54, 3075 | 51·3 | 55 3 | 309·3 | 3·83 | 9·4 | 10·7 | 3 | ·930 |
| 702 | +53, 3263 | 53·1 | 53 44 | 106·9 | 4·27 | 8·9 | 13·3 | 3 | ·869 |
| | +53, 3266 | 54·2 | 53 19 | 105·6 | 16·99 | 8·2 | 9·0 | 2 | ·855 |
| 703 | +53, 3268 | 54·5 | 53 26 | 269·1 | 7·05 | 8·1 | 11·7 | 3 | ·768 AB |
| | | | | 267·2 | 53·03 | C= | 9·5 | 1 | ·671 AC |
| 704 | | 55·0 | 53 27 | 119·4 | 5·45 | 9·5 | 11·5 | 2 | ·709 |

*Notes.*

624.—*h's* angle for AC is 148°·4; if this is correct the close pair may have considerable proper motion.

630.—Angles discordant, difficult in twilight.

648.—Discordant measures.

657.—The p. star of two.

671.—In low-powered field S of $\beta$ 250.

+47°, 3720 A 12 mag. sp. from B.

691.—The nearer *comes*, and the distant one not given by *h*.

697.—On two nights A was suspected to be a close double.

*A new Dividing Engine.* By G. T. McCaw, M.A.
(Plate 13.)

About four years ago a dividing engine was constructed in South London by Messrs. E. R. Watts & Son. The constructors recently submitted a 55 cm. circle divided on this machine to the Charlottenburg Technical Institute. The result of this test may not be without interest to many members of the Society.

A few details of the machine may be given.

The main frame is of cast iron, while the revolving table is of phosphor bronze; the central spindle is of phosphor bronze, fitted in collars of oil-hardened steel. The fitting of the latter was done at the Whitworth Works, and the resultant eccentricity was subsequently found to be exceedingly small; all the rest of the machine was constructed by the owners themselves. The revolving table weighs about five cwt., and the whole engine two tons.

After the revolving table was in position, three silver rings were inlaid on the circumference of the upper face. The circle was first hand-divided, and then this graduation was tested with seven micrometers. The resultant errors were used for recutting the *traits*, and these were again tested for error. This adjusting process was continuously repeated for five months, until the micrometers indicated that none of the 4320 *traits* exhibited a greater error that 0″·6.

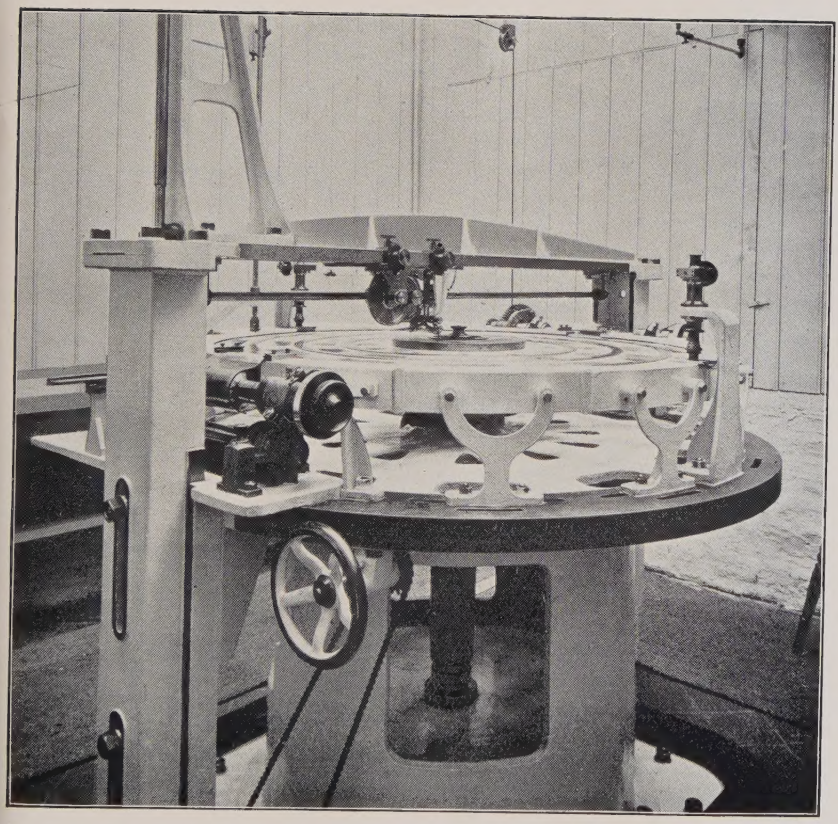
This graduation was now utilised for cutting the peripheral teeth of the revolving table, which engage the endless screw of the rotating gear. Every tooth was cut only after the V-cutter had been adjusted from the mean reading of the seven micrometers. Under all circumstances the temperature is kept as constant as possible, the engine being placed in a specially constructed room.

The engine is operated from a public electrical supply, and in cutting ordinary small circles for theodolites the working is entirely automatic. For the division of finer circles, the work is carried on during those periods of the day when the voltage is nearly constant, as changes of potential would slightly affect the accuracy of the graduation.

The engine will divide circles of from 3 in. to 4 ft. 6 in. diameter. The range of graduation extends from less than 1 minute up to 30 minutes of arc. Unlike some other constructors, this firm believes in dividing continuously; they are of opinion that the other system would involve more risk of error, due to local changes of temperature in the graduating-room.

The dividing can be controlled cut by cut, so that if slight inaccuracies in the peripheral teeth or endless screw or other part of the machine are proved productive of regular errors at certain parts of the circle, these errors may be mechanically eliminated.

The results of the examination at Charlottenburg are given in the table below. In this table the figures are so arranged that the sum of the errors of the thirty divisional lines, the numbers of which are successive multiples of 12, is zero. These thirty lines



CIRCULAR AUTOMATIC DIVIDING ENGINE.

| Deg. | Error. | Deg. | Error. | Deg. | Error. | Deg. | Error. | Deg. | Error. | Deg. | Error. | Deg. | Error. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ″ | | ″ | | ″ | | ″ | | ″ | | ″ | | ″ |
| 0 | +0·1 | 52 | −0·4 | 104 | +0·1 | 156 | −0·3 | 208 | +0·6 | 260 | −1·0 | 312 | −0·2 |
| 1 | 0·0 | 53 | −0·8 | 105 | −0·1 | 157 | −0·4 | 209 | +1·0 | 261 | −1·5 | 313 | −0·2 |
| 2 | 0·0 | 54 | −0·5 | 106 | −0·1 | 158 | −0·2 | 210 | +0·9 | 262 | −1·0 | 314 | −0·3 |
| 3 | +0·3 | 55 | −0·1 | 107 | +0·2 | 159 | −0·1 | 211 | +1·1 | 263 | −1·7 | 315 | −0·6 |
| 4 | +0·1 | 56 | −0·3 | 108 | 0·0 | 160 | −0·2 | 212 | +0·9 | 264 | −0·8 | 316 | −0·4 |
| 5 | +0·5 | 57 | −0·6 | 109 | +0·4 | 161 | +0·1 | 213 | +1·2 | 265 | −0·4 | 317 | −0·7 |
| 6 | +0·3 | 58 | −0·5 | 110 | +0·7 | 162 | −0·1 | 214 | +0·3 | 266 | −0·2 | 318 | −1·0 |
| 7 | +0·1 | 59 | +0·4 | 111 | +0·4 | 163 | −0·4 | 215 | +1·5 | 267 | −0·7 | 319 | −0·5 |
| 8 | +0·5 | 60 | −0·2 | 112 | 0·0 | 164 | −0·7 | 216 | +0·6 | 268 | −0·5 | 320 | −0·5 |
| 9 | +0·4 | 61 | −0·2 | 113 | −0·2 | 165 | −1·3 | 217 | +0·6 | 269 | −0·1 | 321 | −0·5 |
| 10 | −0·6 | 62 | −0·1 | 114 | +0·2 | 166 | −1·2 | 218 | +0·5 | 270 | −0·2 | 322 | −1·0 |
| 11 | +0·3 | 63 | −0·4 | 115 | −0·2 | 167 | −1·1 | 219 | +0·8 | 271 | −0·3 | 323 | −1·2 |
| 12 | +0·1 | 64 | −0·4 | 116 | 0·0 | 168 | −1·6 | 220 | +1·1 | 272 | +0·1 | 324 | −0·8 |
| 13 | 0·0 | 65 | −0·2 | 117 | −0·2 | 169 | −1·6 | 221 | +0·2 | 273 | −0·2 | 325 | −1·0 |
| 14 | −0·2 | 66 | +0·3 | 118 | +0·2 | 170 | −1·0 | 222 | +0·7 | 274 | +0·2 | 326 | −0·6 |
| 15 | −0·5 | 67 | +0·5 | 119 | +0·1 | 171 | −1·6 | 223 | +1·0 | 275 | +0·4 | 327 | −0·7 |
| 16 | −0·3 | 68 | −0·4 | 120 | 0·0 | 172 | −0·7 | 224 | +0·5 | 276 | +0·4 | 328 | −0·8 |
| 17 | 0·0 | 69 | −0·5 | 121 | +0·3 | 173 | −1·0 | 225 | +0·7 | 277 | −0·3 | 329 | −0·7 |
| 18 | −0·2 | 70 | −0·1 | 122 | +0·4 | 174 | −1·1 | 226 | +0·2 | 278 | +0·5 | 330 | −0·9 |
| 19 | −0·1 | 71 | −0·3 | 123 | +0·2 | 175 | −1·3 | 227 | +0·2 | 279 | +0·3 | 331 | 0·0 |
| 20 | −0·2 | 72 | −0·3 | 124 | +0·3 | 176 | −1·0 | 228 | +0·5 | 280 | 0·0 | 332 | −0·5 |
| 21 | +0·2 | 73 | +0·2 | 125 | 0·0 | 177 | 0·0 | 229 | +0·6 | 281 | +0·2 | 333 | −0·8 |
| 22 | −0·1 | 74 | +0·5 | 126 | +0·1 | 178 | 0·0 | 230 | +0·3 | 282 | −0·5 | 334 | 0·0 |
| 23 | +0·1 | 75 | +0·4 | 127 | +0·1 | 179 | +0·3 | 231 | +0·6 | 283 | −0·3 | 335 | −0·3 |
| 24 | +0·1 | 76 | +0·3 | 128 | +0·2 | 180 | 0·0 | 232 | +0·1 | 284 | −0·8 | 336 | −0·4 |
| 25 | 0·0 | 77 | +0·6 | 129 | +0·5 | 181 | −0·5 | 233 | 0·0 | 285 | −0·8 | 337 | +0·3 |
| 26 | +0·1 | 78 | +0·6 | 130 | +0·5 | 182 | −0·6 | 234 | 0·0 | 286 | −0·2 | 338 | +0·5 |
| 27 | +0·5 | 79 | +0·9 | 131 | −0·1 | 183 | −0·7 | 235 | −0·2 | 287 | 0·0 | 339 | +0·7 |
| 28 | +0·7 | 80 | +0·5 | 132 | −0·1 | 184 | −0·8 | 236 | −0·1 | 288 | +0·2 | 340 | −0·1 |
| 29 | +0·7 | 81 | +0·3 | 133 | −0·1 | 185 | −0·4 | 237 | −0·2 | 289 | −0·3 | 341 | +0·1 |
| 30 | +0·5 | 82 | +0·6 | 134 | −0·5 | 186 | −0·6 | 238 | +0·3 | 290 | −0·5 | 342 | +0·2 |
| 31 | +0·5 | 83 | +0·3 | 135 | −1·0 | 187 | −0·6 | 239 | +1·0 | 291 | −0·6 | 343 | −0·5 |
| 32 | +0·1 | 84 | −0·2 | 136 | −0·9 | 188 | +0·2 | 240 | +0·4 | 292 | −0·7 | 344 | +0·1 |
| 33 | +0·3 | 85 | −0·1 | 137 | −0·5 | 189 | 0·0 | 241 | +0·7 | 293 | −0·2 | 345 | −0·7 |
| 34 | +1·3 | 86 | −0·1 | 138 | −0·9 | 190 | −0·5 | 242 | +0·9 | 294 | −0·2 | 346 | −0·9 |
| 35 | +1·4 | 87 | −0·3 | 139 | −0·9 | 191 | −0·1 | 243 | +1·3 | 295 | +0·1 | 347 | −0·4 |
| 36 | +1·3 | 88 | +0·4 | 140 | −0·6 | 192 | −0·1 | 244 | +0·7 | 296 | −0·4 | 348 | −0·8 |
| 37 | +1·8 | 89 | 0·0 | 141 | −0·7 | 193 | −0·9 | 245 | +1·0 | 297 | −0·4 | 349 | −0·8 |
| 38 | +1·5 | 90 | −0·3 | 142 | −1·0 | 194 | −0·6 | 246 | +1·5 | 298 | −0·9 | 350 | −0·7 |
| 39 | +1·3 | 91 | −0·3 | 143 | −0·5 | 195 | −0·7 | 247 | +0·8 | 299 | −1·0 | 351 | −0·7 |
| 40 | +0·9 | 92 | +0·2 | 144 | −0·4 | 196 | −0·6 | 248 | +0·8 | 300 | −0·5 | 352 | +0·4 |
| 41 | +0·4 | 93 | +1·0 | 145 | −0·3 | 197 | −0·2 | 249 | +1·1 | 301 | −0·3 | 353 | +0·2 |
| 42 | +0·6 | 94 | +0·9 | 146 | −0·3 | 198 | 0·0 | 250 | +1·5 | 302 | −0·7 | 354 | +0·6 |
| 43 | +0·5 | 95 | +0·8 | 147 | +0·2 | 199 | −0·2 | 251 | +1·5 | 303 | −0·8 | 355 | +0·2 |
| 44 | +0·6 | 96 | +0·6 | 148 | −0·4 | 200 | +0·1 | 252 | +1·1 | 304 | −1·0 | 356 | −0·2 |
| 45 | +0·5 | 97 | +0·1 | 149 | −0·3 | 201 | −0·2 | 253 | +0·4 | 305 | −0·6 | 357 | +0·3 |
| 46 | +0·6 | 98 | +0·4 | 150 | −0·6 | 202 | −0·2 | 254 | +0·5 | 306 | −0·5 | 358 | +0·2 |
| 47 | +0·1 | 99 | +0·6 | 151 | −0·5 | 203 | −0·1 | 255 | +0·8 | 307 | −1·0 | 359 | +0·4 |
| 48 | +0·7 | 100 | +0·2 | 152 | +0·1 | 204 | +0·6 | 256 | +0·4 | 308 | −1·1 | 360 | +0·1 |
| 49 | +0·2 | 101 | 0·0 | 153 | −0·1 | 205 | +0·7 | 257 | +1·0 | 309 | −1·0 | | |
| 50 | 0·0 | 102 | 0·0 | 154 | −0·1 | 206 | +0·3 | 258 | −0·1 | 310 | −0·2 | | |
| 51 | −0·5 | 103 | +0·2 | 155 | −0·8 | 207 | +1·3 | 259 | −0·4 | 311 | −0·8 | | |

form the basis for the fundamental examination, and the remaining values are inserted between them.

The figure-values indicated are reliable for the thirty fundamental lines to about $\pm 0''{\cdot}3$, for the remaining lines in the average to about $\pm 0''{\cdot}5$.

An analysis of the table of error shows that the range is $+1''{\cdot}8$ to $-1''{\cdot}7$. From 0° to 180°, only 14 errors exceed 1″; from 180° to 360°, 16. The first 180° contain 50 errors greater than p.e. of observation; the second 180°, 90 errors of this kind; in all, 39% of the errors are less than the p.e. of observation. As regards sign, 155 are *plus* and 180 *minus*.

These results seem to indicate that the limit of accuracy is closely approached; yet we understand the constructors have in view new schemes for improving the division. The greater errors appear in seven well-defined groups, and it is to the elimination of these greater errors that attention should be directed, as most of the others are either admittedly within the p.e. of observation, or so very nearly so that their removal would be barely possible.